THE UNIVERSITY OF ALBERTA

AN EPISTEMOLOGICAL ANALYSIS

OF THE

EINSTEIN, PODOLSKY ROSEN PARADOX

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES AND RESEARCH
IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR
THE DEGREE OF MASTER OF ARTS

DEPARTMENT OF PHILOSOPHY

by

© HERBERT KORTE

EDMONTON, ALBERTA

FALL, 1971

THE UNIVERSITY OF ALBERTA

FACULTY OF GRADIATE STUDIES AND RESEARCH

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research for acceptance a thesis entitled An Epistemological Analysis of the Einstein, Podolsky Rosen Paradox submitted by Herbert Korte in partial fulfilment of the requirements for the degree of Master of Arts.

ABSTRACT

The Einstein-Podolsky-Rosen Paradox is stated and analyzed and a critical analysis of some of the different interpretations that resulted from it is given.

Particular emphasis is put on Furry's response to the Einstein-Podolsky-Rosen Paradox. Furry points out that Einstein, Podolsky and Rosen assume in their paper that a physical system has indepentently real properties as soon as it is freed from any kind of physical interference. He draws attention to the fact that this assumption (i.e. Assumption A), is based on a classical concept of physical reality. He shows that certain results of quantum mechanics cannot be reconciled with this assumption, and that in certain physical situations such an assumption leads to results which conflict with quantum mechanics.

It is argued that Furry's formal demonstration that Assumption A is actually untrue is based on the faith that the postulates of quantum theory hold in a hypothetical situation as discussed by Einstein, Podolsky and Rosen. A recent paper by Bohm and Aharanov in which it is claimed that an experiment of Wu and Shaknov can be considered as empirical evidence against Assumption A is then discussed. Assumption A is restated in terms of the Spin problem and an analysis of the empirical and logical relationship between Assumption A and Einstein, Podolsky and Rosen's criterion of physical reality (i.e. C_b) is given.

An examination of the logical relationship between Assumption A and C_b reveals that Assumption A is entailed by C_b. It follows that if Assumption A is untrue and must be rejected if quantum theory is correct, then C_b must also be rejected.

Moreover, attention is drawn to the fact that it is on the basis of experimental results and on that basis alone, that Assumption A and consequently C_b is to be abandoned.

However, since the correctness of a theory does not necessarily entail its completeness the question as to whether or not quantum theory is complete, is still left open.

It is further shown, that the orthodox interpretation of quantum theory is forced to adopt a new epistemological standpoint which compels it to consider the wave function as a complete description of physical reality. Therefore within the conceptual framework of this orthodox interpretation, quantum mechanical description of physical reality is assumed to be complete or as complete as it can ever be.

But it is precisely this assumption which Einstein, Podolsky and Rosen find objectionable. Moreover, this assumption is trans-empirical in nature since it rests on the Phenomenalist's dogma that what is real is only observation and measurement. Therefore the truth or falsehood of this assumption can only be investigated by going outside the conceptual framework of the orthodox interpretation of quantum theory. This Bohm, Vigier and others attempt to do by seeking another interpretation of quantum theory in terms of hidden variable theories.

A qualitative account of Bohm's interpretation of quantum theory in terms of hidden variables is given. His methodological and epistemological reasons for attempting such an interpretation are given support on the basis of a metaphysical analysis of dispositional properties of matter. It is demonstrated that a Realist's account as opposed to a Phenomenalist's account of dispositional properties represents a good working hypothesis for scientific research. It supports the methodology behind the line of research undertaken

by Bohm and others who find the orthodox interpretation of quantum theory inadequate for physical, methodological as well as epistemological reasons and therefore seek a hidden variable interpretation of quantum theory.

ACKNOWLEDGEMENT

I wish to thank Professors C. G. Morgan, M. H. Kelly and D. Sharp for their comments and criticism. I especially wish to thank Dr. A. Z. Capri, Department of Physics, for his generous advice and guiding criticism. Thanks also to Jean for her unfailing support and assistance.

T A B L E   O F   C O N T E N T S

CHAPTER I

AN EXPOSITION OF THE EINSTEIN-PODOLSKY-ROSEN PARADOX

1. Introductory Remarks

Einstein, Podolsky, and Rosen (hereafter referred to as E.P.R.) argue that quantum mechanics does not provide a complete description of physical reality. (1) Their argument rests on several basic metaphysical-epistemological assumptions about physical reality and physical theories, which we shall summarize below. The aim of this chapter is to give an exposition of their arguments.

2. Summary of E.P.R.'s Metaphysical-Epistemological Assumptions And Criteria of a Physical Theory(2)

A. (a) Objective Reality is defined as independent from any theory and the physical concepts with which the theory operates.

(b) The physical concepts of a theory are intended to correspond to Objective Reality.

(c) It is by means of these physical concepts that we picture Objective Reality to ourselves.

B. Success of a physical theory

(a) Success of a physical theory may be judged by:

(i) its correctness

(ii) its completeness

(b) Only if a theory is both correct and complete can it be said that the concepts of the theory are satisfactory.

(c) Correctness is judged by the degree of agreement between the conclusions of the theory and human experience, which in physics takes the form of experiments and measurements.

C. "Necessary Requirement for a complete theory" and a "Sufficient Condition (restricted) of Physical Reality"

(a) Condition of Completeness: Every element of the Physical Reality must have a counterpart in the Physical Theory.

(b) Criterion of a Sufficient Condition of Physical Reality: If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a Physical Quantity, then there exists an element of Physical Reality corresponding to this Physical Quantity.

3. General Remarks About Measurement and Eigenfunctions

Let the wave function ψ completely characterise the state of a given system, ψ being a function of chosen variables that describe a particles' bahaviour. Then, there corresponds to any observable physical quantity A (e.g. momentum) an Hermitian operator $\mathring{A}$, and a set of eigenfunctions ψ_{a_i} each of which represents a normalized state of the system in which the observable quantity A has respectively the precise, real value a_i. More generally, for any n, the expectation value of $\mathring{A}^n$ in state ψ_a is given by

$$\langle \psi_a | \mathring{A}^n | \psi_a \rangle = a^n \qquad (1)$$

For n = 1 we simply get the eigenvalue-equation

$$\mathring{A}\, \psi_a = a\, \psi_a \qquad (2)$$

in which a is the eigenvalue of the operator $\mathring{A}$ and ψ_a is called the eigenfunction of the Physical Quantity A corresponding to the eigenvalue a.

E.P.R. then assert that if the eigenvalue-equation (2) holds for a particle being in an eigenstate ψ_a, then according to C(b) "there is an element of Physical Reality corresponding to the Physical Quantity A," since then the Physical Quantity A has with certainty the real numerical value a.

If, for example, a particle is described in Configuration Space by the eigenfunction

$$\psi_{p_0}(x) = e^{\frac{i}{\hbar} p_0 x} \qquad (3)$$

the Hermitian operator $\mathring{A}$ corresponding to the Physical Quantity A (i.e., the momentum p) is given by $\frac{\hbar}{i}\frac{\partial}{\partial x}$.

$$\mathring{A} = \frac{\hbar}{i}\frac{\partial}{\partial x}$$ is a general statement whose

operational content is more generally expressed by

$$\mathring{A}^n \psi(x) \equiv \left[\frac{\hbar}{i}\frac{\partial}{\partial x}\right]^n \psi(x) = \left(\frac{\hbar}{i}\right)^n \frac{\partial^n \psi(x)}{\partial x^n} \qquad (4)$$

If, as in our example, the particle has one degree of freedom, operating with $\mathring{A}$ on equation (3) results in the eigenvalue-equation

$$\mathring{A}\,\psi_{p_0}(X) = \frac{\hbar}{i}\frac{\partial}{\partial X}\; exp\left[\frac{i}{\hbar} p_0 X\right] = p_0\,\psi_{p_0} \qquad (5)$$

ψ_{p_0} therefore represents a momentum eigenstate of a particle and for a particle in this state the Physical Quantity A has the value p_o.

The position of the particle in Configuration Space is just represented through the independent numerical position variable X. When an observable, such as in our case has the momentum p, is given a definite value (i.e., p_o), we already know from the Indeterminacy Principle, that the value of the position variable X must become completely indefinite.

> In accordance with quantum mechanics we can only say that the relative probability that a measurement of the coordinate will give a result lying between a and b is
>
> $$P(a,b) = \int_a^b \psi^* \psi\, dx = \int_a^b dx = b - a$$
>
> Since this probability is independent of a, but depends only upon the difference b - a, we see that all values of the coordinate are equally probable. (3)

Moreover, the eigenfunction of the operator Q representing the independent variable X in equation (3) cannot be a continuous function of X. The eigenvalue-equation in which a smooth function of X if multiplied by a variable number Q = x does not hold, i.e.

$$Q\,\psi(X) = x\,\psi(X) \neq q\,\psi(X) \qquad (6)$$

What we need is a function that is multiplied by the same number regardless of the value of X, such that

$$x\,\psi' = q\,\psi' \qquad (7)$$

where q is a constant. The function which satisfies this requirement is the Dirac delta-function $\delta(x-q)$ which is (a) zero everywhere except at X = q and (b) is infinite at this point but approaches infinity in such a way that its integral is unity. i.e.

$$\int_{-\infty}^{\infty} \delta(x-q)\,dx = 1 \tag{8}$$

and

$$\delta(x-q) \equiv \int_{-\infty}^{\infty} e^{ik(x-q)}\,dk \tag{9}$$

where $k = \frac{p}{\hbar}$

From equation (8) it follows that

$$f(q) = \int_{-\infty}^{\infty} f(x)\,\delta(x-q)\,dx$$

where f(x) is an arbitrary and continuous function.

$\psi_{p_0}(x)$ therefore cannot give us any information about the position of the particle. Position measurement will disturb the eigenstate $\psi_{p_0}(x)$ resulting in different state ψ' . More generally, if ψ is not an eigenfunction of a Hermitian operator $\overset{o}{A}$, then the value of the <u>Physical Quantity</u> A must show some fluctuations, which are often measured in terms of the mean square of the deviations of the actual value from the mean. i.e.,

$$\langle A^2\rangle - \langle A\rangle^2 = \int \psi^*(x)\left[A^2 - \langle A\rangle^2\right]\psi(x)\,dx \tag{11}$$

or, if A is the <u>Physical Quantity</u>--position

$$\int_{-\infty}^{\infty} \psi^*(x)\left[x - \langle x\rangle\right]^2 \psi(x)\,dx = (\Delta x)^2 \tag{12}$$

The fluctuation is zero when we choose ψ such that

$$A\psi_a = a\psi_a$$ where ψ_a is an eigenfunction.

This reflects the basic principle of the conventional interpretation of quantum mechanics, that when, for example, the momentum of a particle is known, its position coordinate has no Physical Reality. This is more generally expressed through the minimum Indeterminacy Relationship between any two Hermitian operators, say Q and P

$$\langle \Delta Q^2 \rangle \langle \Delta P^2 \rangle \cong \langle \tfrac{i}{2}(QP - PQ) \rangle^2 \qquad (13)$$

or

$$(\Delta Q)^2 (\Delta P)^2 \geq \tfrac{1}{4} \left| \langle (Q, P) \rangle \right|^2 \qquad (13a)$$

This relation results from the non-commutativity of the canonically conjugate variables, say p and x. More generally letting p_i, x_i, respectively be the momentum and position of the i^{th} particle, we can write:

$$(p_i, x_j) = \frac{\hbar}{i} \delta_{ij} \qquad (14)$$

where

$$(p_i, x_j) \equiv (p_i x_j - x_j p_i) \qquad (14a)$$

and where

$$\delta_{ij} \equiv \begin{Bmatrix} 0 & ; & i \neq j \\ 1 & ; & i = j \end{Bmatrix} \qquad (14b)$$

δ_{ij} is defined in this way to indicate that the dynamical variables of different particles commute with each other, since each particle represents totally different degrees of freedom, and thus non-interfering

observables.

In Configuration Space, the momentum operator is defined as

$$P_i = \frac{\hbar}{i}\frac{\partial}{\partial x_i}$$

and the x_i's are mere numbers.

In Momentum Space the p_i's are mere numbers and the position operator is defined as

$$X_i = -\frac{\hbar}{i}\frac{\partial}{\partial p_i} \quad . \quad (4)$$

However, for a structureless particle in one dimension, we simply can express the commutation relation as

$$(P, Q) = \frac{\hbar}{i} \quad ; \quad PQ \neq QP \qquad (15)$$

Non-commutativity therefore means that independent non-interfering observations are not possible, and this implies that the <u>order</u> in which measurements are made is <u>not</u> irrelevant.

From this it follows according to E.P.R. that:

Either M: the quantum mechanical description of <u>Physical Reality</u> given by the wave function is <u>not</u> complete.

Or N: When the operators corresponding to two <u>Physical Quantities</u> do not commute the two quantities <u>cannot</u> have simultaneous <u>Physical Reality</u>.

E.P.R. then construct a hypothetical experiment on the basis of which they claim to have proven $\neg$ N.

$\neg$ N: It is not the case that when operators corresponding to two <u>Physical Quantities</u> do not commute, the two quantities <u>cannot</u> have simultanious <u>Physical Reality</u>.

Therefore if $\neg N$ is true then the two non-commuting <u>Physical Quantities</u> would enter into a <u>complete</u> description of <u>Physical Reality</u> according to the condition of completeness A(a). It follows that, $\neg M$ would imply that these values are contained in the function and are predictable.

But Quantum Theory usually assumes the following to be true:

$$(\neg M \wedge N) \qquad (=_{df} L) \tag{16}$$

Starting then, with the assumption that $\neg M$ is the case, E.P.R. claim to have proven $\neg N$ to be true, which leads to $\neg L$, resulting in the contradiction:

$$\begin{gathered}(\neg M \wedge N) \wedge (\neg M \supset \neg N) \\ (\neg M \wedge N) \wedge \neg(\neg M \wedge N) \\ L \wedge \neg L\end{gathered} \tag{17}$$

4. <u>The Hypothetical Experiment</u>

Let systems S_1 and S_2 (uncorrelated before $t = 0$) interact from $t = 0$ to $t = T$, after which they cease to interact. By stipulation, the states of S_1 and S_2 are known prior to $t = 0$. Therefore we can calculate with Schrödinger's equation the state of the correlated Systems at $t > T$. If the states at $t < T$ were correlated for whatever reason, then these correlations persist in the state function for all time. In our example, the correlation can be calculated from the states of S_1 and S_2 prior to the interaction, interaction of S_1 and S_2, and the absence of interaction at $t > T$.

After interaction has taken place, the combined System S_1 and S_2 can then be expressed by means of the biorthogonal expansion

$$\Psi(x_1, x_2) = \sum_{n=1}^{\infty} \psi_n(x_2)\, U_n(x_1) \tag{18}$$

where $U_n(x_1)$ are the eigenfunctions of a Hermitian operator P_1 representing the observable p_1 in S_1.

Suppose that a measurement is performed on S_1, i.e., we operate on S_1 with $P_1 = \frac{\hbar}{i}\frac{\partial}{\partial x_1}$ and the measurement yields the eigenvalue p_{1k} .

Then, on the basis of the principle known as the "reduction of the wave packet", $U_n(x_1)$ is left in the state $U_k(x_1)$, and $\psi_k(x_2)$ describes the state of S_2.

If the Hermitian operator P_1 possesses a continuous set of eigenvalues, equation (18) should be replaced by an integral

$$\Psi(x_1, x_2) = \int_{-\infty}^{\infty} \psi_p(x_2)\, U_p(x_1)\, dp \tag{19}$$

where the subscript "p" now denotes the continuous eigenvalue spectrum of P_1.

Likewise, if we were to operate on S_1 with operator Q_1 representing another Physical Quantity, and Q_1 too possess a continuous set of eigenvalues q, we would get

$$\Psi(x_1, x_2) = \int_{-\infty}^{\infty} \psi_q(x_2)\, U_q(x_1)\, dq \tag{20}$$

We see that as a consequence of two different measurements performed upon S_1, S_2 is left with two different wave functions $\psi_k(x_2)$ and $\psi_q(x_2)$. But the measurements were performed at $t > T$ i.e. at a time when S_1 and S_2 have ceased to interact and are

merely correlated Systems. In order to avoid "causal anomaly" we are led to conclude that the measurements on $S_1 \big|_{t>T}$ cannot produce any physical changes in S_2. Consequently $\psi_k(x_2)$ and $\psi_q(x_2)$ must refer to the same <u>Physical Reality</u> namely $S_2 \big|_{t>T}$ although they are two different wave-functions.

Moreover, $\psi_k(x_2)$ and $\psi_q(x_2)$ may happen to be eigenfunctions of two non-commuting operators. This would be the case if, for example, we let Q_1 and P_1 be the operators of S_1 representing the <u>Physical Quantities</u> of position and momentum respectively.

For example, the combined System S_1 and S_2 may be composed of two particles whose position coordinates are x_1 and x_2 respectively. If

$$\Psi(x_1, x_2) = \int_{-\infty}^{\infty} \exp\left[\frac{i}{\hbar}(x_1 - x_2 + x_0)p\right] dp \qquad (21)$$

where x_0 is an arbitrary constant, we may choose P_1 as the operator representing the momentum of the first particle. Since

$$U_p(x_1) = \exp\left[\frac{i}{\hbar} p x_1\right] \qquad (22)$$

is an eigenfunction of P_1 corresponding to the eigenvalue p, this implies that

$$\psi_p(x_2) = \exp\left[-\frac{i}{\hbar}(x_2 - x_0)p\right] \qquad (23)$$

describes the eigenstate of S_2, if the operation with P_1 on S_1 yielded the eigenvalue p. The right hand side of equation (23) is now the eigenfunction of the operator $P_2 = \frac{\hbar}{i}\frac{\partial}{\partial x_2}$ corresponding to the eigenvalus -p of the momentum of S_2.

On the other hand if we choose the position coordinate of S_1 as Q_1, its eigenfunction according to previous discussion and equations must have the following form:

$$U_x(X_1) = \delta(X_1 - X) \tag{24}$$

where the eigenfunction $U_x(X_1)$ corresponds to the eigenvalue x. Whence

$$Q_1 \delta(X_1 - X) = X \delta(X_1 - X) \tag{25}$$

and we get

$$\Psi(X_1, X_2) = \int_{-\infty}^{\infty} \psi_x(X_2) U_x(X_1) \, dx \tag{26}$$

or

$$\Psi(X_1, X_2) = \int_{-\infty}^{\infty} \psi_x(X_2) \, \delta(X_1 - X) \, dx \tag{27}$$

From this it follows that according to equation (10)

$$\psi_x(X_2) = \Psi(X, X_2) = \int_{-\infty}^{\infty} \exp\left[\frac{i}{\hbar}(X - X_2 + X_0)p\right] dp \tag{28}$$

Substituting equation (28) into equation (27) we get

$$\Psi(X_1, X_2) = \iint_{-\infty}^{\infty} \left\{ \exp\left[\frac{i}{\hbar}(X - X_2 + X_0)p\right] \delta(X_1 - X) \, dx \right\} dp \tag{29}$$

Since

$$\psi_x(X_2) = \int_{-\infty}^{\infty} \exp\left[\frac{i}{\hbar}(X - X_2 + X_0)p\right] dp = h \, \delta(X - X_2 + X_0) \tag{30}$$

we can now write equation (29) as

$$\Psi(x_1, x_2) = h\int_{-\infty}^{\infty} \delta(x - x_2 + x_0)\,\delta(x_1 - x)\,dx \tag{31}$$

Therefore

$$\psi_x(x_2) = \Psi(x, x_2) = h\,\delta(x - x_2 + x_0) \tag{32}$$

describes S_2 <u>if</u> a measurement of position in S_1 yielded the value x.

But

$\psi_x(x_2) = h\,\delta(x - x_2 + x_0)$ is the eigenfunction of

$$Q_2 = x_2 \tag{33}$$

corresponding to the eigenvalue $x + x_0$, i.e. the position coordinate of S_2.

Thus, although Q_2 and P_2 do not commute, it was possible, without in any way disturbing S_2, to deduce the <u>precise</u> values of the non-commutative <u>Physical Quantities</u>, the position <u>and</u> momentum of S_2.

Therefore, according to E.P.R.'s criterion of <u>Physical Reality</u>, there exists an element of <u>Physical Reality</u> in S_2 which corresponds to the exactly calculated values of position and momentum of S_2.

In fact, these values of S_2 must have existed just prior to the measurements of the values (position and momentum) of S_1, since in view of the absence of any interaction at $t > T$, the measurements on S_1 could not have affected S_2 in any way. To assert the contrary would lead to a "causal anomaly". Thus $\neg N$ is claimed to be proven.

It is clear however, that we cannot ever simultaneously learn of the values of position and momentum of S_2, since to do so would require us to measure the momentum and position of S_1 simultaneously.

Therefore, on the basis of their own assumptions and criteria E.P.R. do not violate the Indeterminacy Principle, and they argue that one need not insist that

> O: two or more Physical Quantities can be regarded as simultaneous elements of Physical Reality <u>only when they can be simultaneously measured or predicted.</u> (5)

If one would insist on O one would make the <u>Physical Reality</u> of such <u>Physical Quantities</u> as position and momentum of S_2 dependent on the measurement process of S_1 itself. But any operation on S_1 at $t > T$ cannot physically affect S_2. Therefore the deduced values of momentum and position <u>must</u> be <u>simultaneously</u> present in S_2 and constitute <u>Physical Reality</u> according to C(b) .

CHAPTER II

EXPOSITION OF FURRY'S RESPONSE TO E.P.R.

1. Introductory Remarks

Furry's paper of November 1935 (1) is a response to E.P.R.'s paper of March of that year. Furry points out that in their paper, E.P.R. assume that a physical system has independently real properties as soon as it is freed from any kind of physical interference. He draws attention to the fact that this assumption is based on a classical concept of physical reality. He shows that certain results of quantum mechanics cannot be reconciled with this assumption, and that in certain physical situations such an assumption leads to results which conflict with quantum mechanics.

To discuss Furry's paper it is convenient to restate his results in the language of state vectors and statistical operators in Hilbert Space.

2. The Statistical Statements of Quantum Theory

In quantum theory, the states of a system S are represented by state vectors $(|\phi\rangle, |\varphi\rangle, |\chi\rangle, \ldots)$ in Hilbert space. Physical quantities of variables--that is measurable quantities--are represented through Hermitian operators $\mathcal{A}, \mathcal{B}, \ldots \mathcal{L} \ldots$ of the Hilbert space H^S associated with the system S. Moreover, a set of vectors $\{|\phi_n\rangle\}$ is orthonormal if and only if $\langle \phi_{n'} | \phi_n \rangle = \delta_{n'n}$; and a set of orthonormal vectors is also complete if and only if for any vector in Hilbert space, there exists at least one set of scalars $\{r_n\}$ such that $|\Psi\rangle = \sum_n r_n |\phi_n\rangle$. A set of vectors $\{|\phi_n\rangle\}$ of H which is both orthonormal and complete is called an orthonormal basis set. We shall call such a set a c.o.n. set, where "c.o.n. is an abbreviation for "complete, orthogonal, and normal". Furthermore, in Hilbert space

all c.o.n. sets of vectors contain an infinite number of orthonormal vectors, making Hilbert space infinite dimensional.

Furry distinguishes between two alternative ways of expressing our statistical information about a system:
(a) The expectation values $\langle \mathcal{L} \rangle$ of all observables of a system.
(b) The probability $\Pi(\Lambda)$ that an arbitrary eigenvalue Λ of an arbitrary observable $\mathcal{L}$ will be obtained.

Digression: Some definitions are in order.
A pure state is an eigenstate.
A mixed state is a superposition (linear) of eigenstates.
A statistical mixture or ensemble of states is a collection of states (pure or mixed).
End of Digression

These two types of statistical information may be obtained in two kinds of physical situations in which:

(i) We have an ensemble or statistical mixture of possible state vectors $|\phi_i\rangle$, and ω_i' are the respective probabilities of a system being in states $|\phi_i\rangle$. The weights ω_i' are real and satisfy the conditions $\sum_i \omega_i' = 1$ and $0 \leq \omega_i' \leq 1$. Furthermore, we shall presuppose that the state vectors are normalized; however, their orthogonality will not be presupposed, i.e. $\langle \phi_i | \phi_j \rangle \neq 0$. The statistical state representation is therefore given by the density operator:

$$W^{(i)} = \sum_i \omega_i' |\phi_i\rangle\langle\phi_i| \qquad (34)$$

If the vectors $|\phi_{\delta i}\rangle$ are eigenvectors of some arbitrary operator representing some physical quantity or dynamical variable having eigenvalues $\delta_i (i = 1, 2, \cdots, n)$, then

$$W^{(i)} = \sum_i \omega_{\delta i}' |\phi_{\delta i}\rangle\langle\phi_{\delta i}| \qquad (34a)$$

represents an ensemble of definite eigenstates, and the realization

of each eigenstate $|\phi_{\xi i}\rangle$ in the ensemble of distinct eigenstates is associated with the respective probability $\omega'_{\xi i}$.

(ii) A system's state is expressed through a single state vector $|\phi\rangle$ which is a linear combination of $|\phi_i\rangle$, that is $|\phi\rangle = |\sum_i (\omega_i)^{1/2}\phi_i\rangle$ where $(\omega_i)^{1/2}$ are certain numerical coefficients (in general complex). We have here an application of the principle of superposition. Its applicability, as we shall see, sharply differentiates the probabilistic aspects of quantum theory from those of classical mechanics. The statistical state representation in this case is given by

$$W^{(ii)} = |\textstyle\sum_i (\omega_i)^{1/2}\phi_i\rangle\langle\textstyle\sum_i (\omega_i)^{1/2}\phi_i| \tag{35}$$

which represents each system as being in a <u>definite state</u>.

Let us apply (a)(i), (a)(ii), (b)(i), (b)(ii) to a one component system S.

(a)(i): What is the expectation value $\langle\mathcal{L}\rangle$ of an observable $\mathcal{L}$ in the case of a statistical mixture? According to the statistical statements of quantum theory, $\langle\mathcal{L}\rangle$ for the state $|\phi_i\rangle$ is given by

$$\langle\mathcal{L}\rangle_i = \langle\phi_i|\mathcal{L}\phi_i\rangle \tag{36}$$

These results have to be averaged over the ensemble with the weights ω'_i , so that in total we get for the expectation value

$$\begin{aligned}\langle\mathcal{L}\rangle &= \textstyle\sum_i \omega'_i\langle\mathcal{L}\rangle = \sum_i \omega'_i\,\langle\phi_i|\mathcal{L}\phi_i\rangle \\ \langle\mathcal{L}\rangle &= \textstyle\sum_i \omega'_i\, Tr\left(P_{|\phi_i\rangle}\mathcal{L}\right)\end{aligned} \tag{37}$$

where $\mathcal{P}_{|\phi_i\rangle} \equiv |\phi_i\rangle\langle\phi_i|$

Since $W^{(i)} \equiv \sum_i \omega_i' \mathcal{P}_{|\phi_i\rangle}$ (38)

we can express $\langle\mathcal{L}\rangle$ in terms of the density operator $W^{(i)}$, i.e.

$$\langle\mathcal{L}\rangle^{(i)} = T_r\left(W^{(i)}\mathcal{L}\right) \tag{39}$$

$$\langle\mathcal{L}\rangle^{(i)} = \sum_i \omega_i' \int_{-\infty}^{\infty} \phi_i^* \mathcal{L} \phi_i \, d\tau \tag{40}$$

It is to be noticed, particularly from equations (37) that we take the average over the statistical mixture of the expectation values $\langle\mathcal{L}\rangle_i$, and not over the states $|\phi_i\rangle$. The mixture is therefore an <u>incoherent superposition</u> of <u>pure states</u>. The various $|\phi_i\rangle$ do not interfere with one another.

(a)(ii): The hypothesis of linear superposition states that if ϕ_1 and ϕ_2 and. . . ϕ_i are possible wavefunctions, then any linear combination of them is also a wavefunction. That is, $\phi = \sum_i r_i \phi_i$.

Writing $|\phi\rangle = |\sum_i (\omega_i)^{1/2}\phi_i\rangle$ we get for the expectation value of an arbitrary operator $\mathcal{L}$ according to equation (35)

$$\langle\mathcal{L}\rangle^{(ii)} = T_r\left(W^{(ii)}\mathcal{L}\right) = \left\langle \sum_i (\omega_i)^{1/2}\phi_i \middle| \mathcal{L} \sum_i (\omega_i)^{1/2}\phi_i \right\rangle \tag{41a}$$

$$= \int_{-\infty}^{\infty} \left[\sum_i (\omega_i)^{1/2}\phi_i\right]^* \mathcal{L} \left[\sum_i (\omega_i)^{1/2}\phi_i\right] d\tau \tag{41b}$$

$$= \int_{-\infty}^{\infty} \phi^* \mathcal{L} \phi \, d\tau \tag{41c}$$

The expectation values $\langle \mathcal{L} \rangle^{(i)}$ and $\langle \mathcal{L} \rangle^{(ii)}$ are not equal due to the interference terms of the probability amplitudes which arise for the statistical operator $W^{(ii)}$, because the superposition principle applies.

(b)(i): On the basis of the above discussion it is easily seen how to obtain the probability of discrete eigenvalues Λ, i.e. $\mathcal{T}(\Lambda)$ for

<u>Case(α):</u> $$W^{(i)}_{pure} \equiv W^{(i)}_{f} = \mathcal{P}_{|\phi\rangle} = \left(\mathcal{P}_{|\phi\rangle}\right)^2 \qquad (42)$$

<u>Case (β):</u> $$W^{(i)}_{mixed} \equiv W^{(i)}_{m} = \sum_i \omega_i' \, \mathcal{P}_{|\phi_i\rangle} \qquad (43)$$

with $0 \leq \omega_i' \leq 1$ and $\sum_i \omega_i' = 1$

Also, $0 \leq \mathcal{T}(\Lambda) \leq 1 \quad ; \quad \forall \Lambda \in \mathcal{L}$

However, before we consider (b)(i)(α) and (b)(i)(β), some <u>very sketchy</u> remarks about the usual interpretation of the measuring process in quantum theory are in order.

In a classical situation the measurement of physical quantities such as, for example, position and momentum, need not, in principle, alter the state of the classical system under observation. It is therefore reasonable to make the ontological assumption that the values of the quantities to be measured exist both before and after the measurement. In principle, at least, a classical measurement on a classical system does not disturb the state of the system. Of course, there are many physical situations where a classical measurement does alter the state of a classical system, and as a

consequence the results of predictive and retrospective measurements will not always coincide. (2) But the distinction between retrospective and predictive measurements in a classical theory is less crucial than in quantum theory, since in a classical situation, the disturbance of the system, as a consequence of the interaction with the measuring device can either be physically and quantitatively accounted for (at least in principle) or made negligible.

Before a quantum mechanical measurement of a physical quantity represented by an Hermitian operator $\mathcal{L}$, one cannot generally say with certainty which particular eigenvalue Λ of the many possible eigenvalues will be obtained, unless one knows before the measurement has taken place, that the state vector has the direction of an eigenvector corresponding to the eigenvalue Λ, that belongs to $\mathcal{L}$. (for sake of simplicity we will usually assume the eigenvalues to be non-degenerate, that is, we will assume that to every eigenvalue belongs exactly one eigenvector.)

If before a measurement, $|\phi\rangle$ lies in the direction $|u_\Lambda\rangle$, then the measurement of $\mathcal{L}$ will give with probability equal to unity (at least in principle--in practice errors of measurements must always be taken into account) the eigenvalue Λ, i.e. $\pi(\Lambda)=1$.

To see this we multiply the eigenvalue equation $\mathcal{L}|u_\Lambda\rangle = \Lambda|u_\Lambda\rangle$ by some arbitrary eigenvector. Since $\mathcal{L}$ is hermitian, i.e. $\mathcal{L}=\mathcal{L}^\dagger$, we get

$$\begin{gathered}\langle u_{\Lambda'}|\mathcal{L}u_\Lambda\rangle = \Lambda\langle u_{\Lambda'}|u_\Lambda\rangle = \\ \langle \mathcal{L}u_{\Lambda'}|u_\Lambda\rangle = \Lambda'^{*}\langle u_{\Lambda'}|u_\Lambda\rangle\end{gathered} \tag{44a}$$

Therefore

$$(\Lambda-\Lambda'^{*})\langle u_{\Lambda'}|u_\Lambda\rangle = 0$$

and $$(\Lambda - \Lambda'^{*}) |\langle u_{\Lambda'} | u_{\Lambda} \rangle|^2 = 0 \tag{44b}$$

If $\Lambda' = \Lambda$ we conclude since $\| u_\Lambda \| = \sqrt{\langle u_\Lambda | u_\Lambda \rangle} \neq 0$ that the eigenvalues of Hermitian operators are real, i.e. $\Lambda = \Lambda^*$, and $|\langle u_{\Lambda'} | u_\Lambda \rangle|^2 = \delta_{\Lambda'\Lambda}$ implies $\Pi(\Lambda) = 1$.

<u>Digression:</u> If the eigenvalue Λ is t_Λ - fold degenerate, then the eigenvalue equation is satisfied for one eigenvalue Λ through several linearly independent eigenvectors $|u_\Lambda^v\rangle$, and

$$\mathcal{L} | u_\Lambda^v \rangle = \Lambda | u_\Lambda^v \rangle \ , \text{ for } v = 1, \ldots, t_\Lambda$$

Then $\langle u_\Lambda^v | u_\Lambda^{v'} \rangle$ is not necessarily equal to $\delta^{vv'}$, but it is always possible for cases of degenerate eigenvalue problems to introduce orthonormal eigenvectors $|\hat{u}_\Lambda^v\rangle$, such that $\langle \hat{u}_\Lambda^v | \hat{u}_{\Lambda'}^{v'} \rangle = \delta_{\Lambda\Lambda'} \delta_{vv'}$ is satisfied by using the Schmidt orthogonalization procedure. (3)

<u>End of Digression</u>

As was pointed out, one can never be sure before the completion of a measurement, whether or not $|\phi\rangle$ lies in the direction of a particular eigenvector $|u_\Lambda\rangle$, (or whether $|\phi\rangle$ lies in Hilbert space $\mathcal{U}_\Lambda$ in case of degeneracy). We can know however with certainty, <u>immediately</u> after a measurement has been completed, and a measured value Λ obtained, that the direction of the state vector <u>now</u> lies in the direction $|u_\Lambda\rangle$.

Suppose <u>immediately</u> after an exact and successful measurement the value Λ' is obtained. Then the initial state vector $|\phi\rangle$ has been transformed in such a way during the interaction between the object and the measuring device, that immediately after such a measurement is completed, $|\phi\rangle$ has been transformed into $|\phi'\rangle = |u_{\Lambda'}\rangle$ Messiah calls this "(non-causal) change of the wavefunction by the measurement process the <u>filtering of the wave packet</u>." (4)

If we imagine the first measurement to be immediately repeated, then it is necessary, if measurements in quantum theory do have any physical meaning at all, that the second measurement yields [at least in principle, c.f.p.19] with certainty, i.e. with $\Pi(\Lambda') = 1$ the value Λ' . (c.f. Chapter IV p. 69 for a fuller discussion of this point)

This implies that an exact and successful measurement of the eigenvalue Λ_i , whether it is a predictive of retrospective measurement, must always result in a numerical value corresponding to one of the possible eigenvalues Λ .

Furthermore, if a successful and exact individual predictive measurement yields Λ_i and is followed immediately by an exact retrospective measurement which yields Λ'_i , then the results of the two measurements must agree, that is, $\Lambda_i = \Lambda'_i$.

The initial statevector will transform upon a successful measurement into the direction of the many possible eigenvectors which belong to $\mathcal{L}$. Which direction cannot be known with certainty according to the usual interpretation of quantum theory because of the essentially uncontrollable interaction between the measuring instrument and the object of measurement. It is a matter of probability statements which can be expressed mathematically as follows:

If we make a large number n of measurements on systems which are prepared so as to be in the same dynamical state at the instant of measurement and record the number $n(\Lambda)$ of these experiments which result in the value Λ , then the probability that Λ will obtain is

$$\Pi(\Lambda) = \lim_{n \to \infty} \frac{n(\Lambda)}{n} \quad . \tag{45}$$

It follows that the expectation value $\langle \mathcal{L} \rangle$ is given by

$$\langle \mathcal{L} \rangle = \lim_{n \to \infty} \frac{1}{n} \sum_{\Lambda} n(\Lambda)\,\Lambda = \sum_{\Lambda} \Pi(\Lambda)\,\Lambda \tag{46}$$

(b)(i): For any observable $\mathcal{L}$ the eigenvectors $|u_\Lambda\rangle$ form a complete set and therefore must be taken as orthonormal.

Case (α): $\Pi(\Lambda) = \langle P_{|u_\Lambda\rangle} \rangle = Tr\left(W_f^{(i)} P_{|u_\Lambda\rangle} \right)$

$$\Pi(\Lambda) = Tr\left(P_{|\phi\rangle} P_{|u_\Lambda\rangle} \right) = |\langle \phi | u_\Lambda \rangle|^2 \tag{47}$$

Case (β): $\Pi(\Lambda) = \langle P_{|u_\Lambda\rangle} \rangle = Tr\left(W_m^{(i)} P_{|u_\Lambda\rangle} \right)$

$$= \sum_i \omega_i' |\langle \phi_i | u_\Lambda \rangle|^2 = \sum_i \omega_i' \Pi_i(\Lambda) \tag{48}$$

where $\Pi_i(\Lambda) \equiv |\langle \phi_i | u_\Lambda \rangle|^2$

Analogous to the previously discussed case in which we took the average over the mixture of the expectation values $\langle \mathcal{L} \rangle_i$ (c.f. equation 37), the quantum-theoretical probabilities $\Pi_i(\Lambda)$ now are averaged over by the weights ω_i' of the mixture.

The above result could also be derived using equation (46), i.e. $\langle \mathcal{L} \rangle = \sum_{\Lambda} \Pi(\Lambda)\,\Lambda$.

For case(β) we reason as follows:

$$\langle \mathcal{L} \rangle = \sum_{\Lambda} \Pi(\Lambda)\,\Lambda = \sum_{i,\Lambda} \omega_i' \Pi_i(\Lambda)\,\Lambda = \sum_i \omega_i' \langle \mathcal{L} \rangle_i$$

Therefore $\langle \mathcal{L} \rangle = \sum_i \omega_i' \langle \phi_i | \mathcal{L}\, \phi_i \rangle$

Since $$1\mathscr{L}1 = \mathscr{L} = \sum_k |u_{\Lambda_k}\rangle\langle u_{\Lambda_k}|\mathscr{L}\sum_j |u_{\Lambda_j}\rangle\langle u_{\Lambda_j}| \qquad (49)$$

$$\langle\mathscr{L}\rangle = \sum_i \omega_i' \sum_{k,j} \langle\phi_i|u_{\Lambda_k}\rangle\langle u_{\Lambda_k}|\mathscr{L}|u_{\Lambda_j}\rangle\langle u_{\Lambda_j}|\phi_i\rangle \qquad (50)$$

$$\langle\mathscr{L}\rangle = \sum_i \omega_i' \sum_{k,j} \langle\phi_i|u_{\Lambda_k}\rangle\langle u_{\Lambda_k}|\Lambda_j|u_{\Lambda_j}\rangle\langle u_{\Lambda_j}|\phi_i\rangle \qquad (51)$$

$$\langle\mathscr{L}\rangle = \sum_i \omega_i \sum_{k,j} \langle\phi_i|u_{\Lambda_k}\rangle \Lambda_j \delta_{jk} \langle u_{\Lambda_j}|\phi_i\rangle$$

$$\langle\mathscr{L}\rangle = \sum_{i,k} \omega_i' |\langle\phi_i|u_{\Lambda_k}\rangle|^2 \Lambda_k \qquad (52)$$

which must be equal to equation (46), i.e. $\langle\mathscr{L}\rangle = \sum_\Lambda \Pi(\Lambda)\Lambda$
It follows immediately that

$$\Pi(\Lambda) = \sum_i \omega_i' |\langle\phi_i|u_\Lambda\rangle|^2$$

Moreover, equation (50) expresses again very clearly the difference between $\langle\mathscr{L}\rangle^{(i)}$ and $\langle\mathscr{L}\rangle^{(ii)}$ (c.f.p.17). The quantum mechanical calculations for expectation values, i.e. $\langle\mathscr{L}\rangle^{(ii)}$, are performed by means of probability amplitudes and lead to interference terms of the latter, whereas the calculations for $\langle\mathscr{L}\rangle^{(i)}$ consist in taking the average over the mixture of the expectation values $\langle\mathscr{L}\rangle_i^{(ii)} = \langle\phi_i|\mathscr{L}\phi_i\rangle$ by means of the weights ω_i' , and do not result in interference terms.

(b)(ii): The density operator is given by

$$\overset{(ii)}{W} = \Big| \sum_i (\omega_i)^{1/2} \phi_i \Big\rangle \Big\langle \sum_i (\omega_i)^{1/2} \phi_i \Big|$$

Therefore

$$\Pi(\wedge) = \langle \mathcal{P}_{|u_\wedge\rangle} \rangle = T_r \big(\overset{(ii)}{W} \mathcal{P}_{|u_\wedge\rangle} \big)$$

$$= \Big| \Big\langle \sum_i (\omega_i)^{1/2} \phi_i \Big| u_\wedge \Big\rangle \Big|^2 \tag{53}$$

3. Reduction of the Wavepacket

Furry states, (referring to von Neumann's Mathematische Grundlagen der Quantenmechanik, p. 167-8) that for any wave function $\Psi(X_1, X_2)$ of two systems S_I and S_{II} "which have at some previous time interacted and have now ceased to interact, there always exists an expansion which is in general unique, in the form

$$\Psi(X_1, X_2) = \sum_K (\omega_k)^{1/2} \varphi_{\lambda k}(X_1)\, \xi_{\rho k}(X_2)." \text{ (5)} \tag{54}$$

In the language of state vectors we write equation (54) as

$$|\Psi^{I,II}\rangle = \sum_K (\omega_k)^{1/2} |\varphi^{I}_{\lambda k}\rangle \otimes |\xi^{II}_{\rho k}\rangle \tag{54a}$$

where $\{|\varphi^I_{\lambda k}\rangle\}$ and $\{|\xi^{II}_{\rho k}\rangle\}$ are sets of c.o.n. eigenstates of S_I (in Hilbert space H^I) and S_{II} (in Hilbert space H^{II}) respectively. Moreover, $|\Psi^{I,II}\rangle$ lies in Hilbert space $H = H^I \otimes H^{II}$. The sets $\{|\varphi^I_{\lambda k}\rangle\}$ and $\{|\xi^{II}_{\rho k}\rangle\}$ are eigenstates of observables $\mathcal{L}$ (corresponding to eigenvalues λ_k) and $\mathcal{R}$ (corresponding to eigenvalues ρ_k) respectively.

According to equations (54) the statistical information we have of S_I and S_{II} is W^I and W^{II} respectively, where

$$Tr^{II}(W^{I+II}) = W^I = \sum_K \omega_K |\varphi^I_{\lambda K}\rangle\langle\varphi^I_{\lambda K}| \tag{55}$$

$$Tr^{I}(W^{I+II}) = W^{II} = \sum_K \omega_K |\xi^{II}_{\rho K}\rangle\langle\xi^{II}_{\rho K}| \tag{56}$$

[Tr^I and Tr^{II} are the operations of taking the trace in H^I and H^{II} respectively].

A measurement of either $\mathcal{L}$ or $\mathcal{R}$ on the composite system S_{I+II} will, according to equation (54), always give us λi of $\mathcal{L}$ corresponding to ρi of $\mathcal{R}$, or ρi of $\mathcal{R}$ corresponding to λi of $\mathcal{L}$. Making therefore a measurement of $\mathcal{L}$ on S_I enables us to predict with certainty the eigenvalue of the observable $\mathcal{R}$ of S_{II} since immediately after the measurement of $\mathcal{L}$

$$\overset{(i)\,I+II}{W_m} = \sum_K \omega_K |\varphi^I_{\lambda K}\rangle\langle\varphi^I_{\lambda K}| \otimes |\xi^{II}_{\rho K}\rangle\langle\xi^{II}_{\rho K}| \tag{57}$$

reduces to

$$\overset{(i)\,I+II}{W_f} = \overset{(i)\,I}{W_f} \otimes \overset{(i)\,II}{W_f} = P_{|\varphi^I_{\lambda i}\rangle} \otimes P_{|\xi^{II}_{\rho i}\rangle} \tag{58}$$

and

$$\Pi(\rho i) = Tr\left(\overset{(i)\,II}{W_f} P_{|\xi^{II}_{\rho i}\rangle}\right) = Tr\left[Tr^I\left(\overset{(i)\,I+II}{W_f}\right) P_{|\xi^{II}_{\rho i}\rangle}\right]$$

$$\Pi(\rho i) = \left(P_{|\xi^{II}_{\rho i}\rangle}\right)^2 = 1 \tag{59}$$

Furry concludes that equation (54) shows that the coupling between S_I and S_{II} has been such as to make S_I a suitable instrument for measuring the observable $\mathcal{R}$ on S_{II}, the physical quantity $\mathcal{L}$ serving as a "pointer reading". (6)

This may mean, for example, that we interpret S_I as a measuring instrument and S_{II} as the object to be measured. Then the interaction between S_I and S_{II} constitutes a measuring process, and the ceasing of this interaction (for example by "disconnecting" or "isolating" the measuring instrument from the object) constitutes in part the completion of the measurement. To complete the measuring process we take a further step, a reading, and the quantity $\mathcal{L}$ of S_I may then serve as a "pointer reading", telling us something about S_{II}.

"In any measuring process actually used," Furry says, "the biorthogonal' expansion . . . [equation (54)] is unique, and the experimenter concerns himself only with the observable and 'pointer reading' which belong to this expansion. This Assumption A [c.f.p 28] is a perfectly safe working hypothesis."(7) However, the contradictions which he wishes to investigate, "can be brought out only by going beyond the considerations given in connection with [equation (54)] . We may either look for particular cases in which the expansion [equation (54)] is not unique--e.g. the example given by E.P.R., . . . or develop a way of interpreting expansions of a less special type." (8)

Now the more general method of "reducing the wave packet" is made use of, for example, in a situation where S_I and S_{II} are two correlated systems that are after some time of interaction isolated (say spatially), such that the interaction between them is negligible and for all purposes zero. But the interaction is such that the state of the composite system may be given by either of at least two alternative expansions of orthogonal functions. Writing these expansions in terms of state vectors in Hilbert space we get

$$|\Psi^{I+II}\rangle = \sum_{\mu} |\psi_{\mu}^{I}\rangle \otimes |\xi_{\mu}^{II}\rangle \tag{60}$$

$$|\Psi^{I+II}\rangle = \sum_{\sigma} |\theta_{\sigma}^{I}\rangle \otimes |\eta_{\sigma}^{II}\rangle \tag{60a}$$

where $|\psi_{\mu}^{I}\rangle$ and $|\xi_{\mu}^{II}\rangle$ may, for example, be eigenfunctions of momentum of S_{I} and S_{II} respectively, corresponding to the observable $\mathcal{M}$, and $|\theta_{\sigma}^{I}\rangle, |\eta_{\sigma}^{II}\rangle$ may be eigenfunctions of position of S_{I} and S_{II} respectively corresponding to the observable S.

In equation (54) $\quad |\xi_{\mu}^{II}\rangle = \langle \psi_{\mu}^{I} | \Psi^{I+II}\rangle \quad$ (61)

is a vector in H^{II}, the Hilbert space associated with S_{II}. Taking the scalar product of both sides of equation (60) we get

$$\langle \psi_{\mu'}^{I} | \Psi\rangle = \sum_{\mu} \langle \psi_{\mu'}^{I} | \psi_{\mu}^{I}\rangle \delta_{\mu'\mu} \otimes |\xi_{\mu}^{II}\rangle \tag{62}$$

Whence it follows that

$$|\xi_{\mu'}^{II}\rangle = \langle \psi_{\mu'}^{I} | \Psi\rangle = \int_{-\infty}^{\infty} \psi_{\mu'}^{*}(x_1)\, \Psi(x_1, x_2)\, dx_1 \tag{63}$$

4. Probability Calculations in Terms of Method A and Method B

Furry distinguishes between what he calls Method A and Method B. He uses these methods to calculate the probabilities of eigenvalues which may be obtained from S_{II} in a few separate cases.

Besides the observables $\mathcal{L}$ and $\mathcal{R}$ (which have a special significance through their connection with equation (54), he considers two other arbitrary observables $(\mathcal{M}, S) \neq (\mathcal{L}, \mathcal{R})$ with eigenvalues μ, σ and corresponding eigenvectors $|\psi_\mu\rangle$, $|\eta_\sigma\rangle$ respectively.

Method A is based on a definite view of physical reality which E.P.R. hold. This view rests on the assumption that once a system is free from any significant mechanical interference, it has independently real properties. Furry formulates this assumption as follows: (the division of points is mine)

Assumption A and Method A

(Ai): We assume that during the interaction of S_I and S_{II} each system made a transition to a definite state in which it now is, S_I being in one of the states $|\varphi^{I}_{\lambda k}\rangle$ and S_{II} in one of the states $|\xi^{II}_{\rho k}\rangle$

(Aj): These transitions are not causally determined, and there is no way of finding out which transition occurred, except by making a suitable measurement. In the absence of measurements we know only that the probabilities of the different transitions are respectively ω_k and that if S_I is in the state $|\varphi_{\lambda i}\rangle$ S_{II} is in the state $|\xi_{\rho i}\rangle$ [c.f. equation (54)]

(Ak): This provides a sufficient basis for making all needed calculations of probabilities, the methods being those of ordinary probability theory. (9)

It is clear that according to Assumption A the statistical state representation must be of the type $W^{(i)}$ (c.f. equation 34a) which represents an ensemble of definite eigenstates. Thus for the composite system S_{I+II}

$$W^{(i)} = W_m^{(i)\,I+II} = \sum_K \omega_K \,|\varphi_{\lambda K}^{I}\rangle\langle\varphi_{\lambda K}^{I}| \otimes |\xi_{\rho K}^{II}\rangle\langle\xi_{\rho K}^{II}| \qquad (64)$$

Method B: Quantum mechanically we reason as follows: Given equation (60), then after a measurement on S_I has given the value μ' for $\mathcal{M}$, S_{II} is in the pure state with state vector given--apart from normalization--by equation (63). Then the probability of finding the eigenvalue δ for some arbitrary observable $\mathcal{F} \neq (\mathcal{L}, \mathcal{R})$ in S_{II}, i.e. $\Pi(\delta\,|\,\mu')$, is given by

$$\frac{|\langle \xi_{\mu'}^{II} | \chi_{\delta}^{II}\rangle|^2}{\langle \xi_{\mu'}^{II} | \xi_{\mu'}^{II}\rangle} \qquad (65)$$

where $|\chi_\delta\rangle$ is the eigenvector of the observable $\mathcal{F}$ corresponding to some physical quantity having eigenvalue δ .

Furry now proceeds to show the agreement and disagreement between Method A based on Assumption A and Method B based on quantum mechanical formalism. For the cases (a),(b),(c) both methods give identical results. However, for case (d) a formal discrepancy arises and is a consequence of the fact that in Method B the superposition principle applies giving rise to interference terms between probability amplitudes.

Case(a): If S, having eigenvalues σ and eigenfunctions η_σ is measured on S_{II} without any measurements having been made on S_I, what is the probability of obtaining the result σ' ? (10)

Since we are only concerned with S_{II}, the statistical information of S_{II}, i.e. $W_m^{(i) II}$ is given, because of equation (54) by equation (57), as a mixture of the states $|\xi^{II}_{\rho k}\rangle$ with weights ω_k . Therefore

$$W_m^{(i) II} = T_r\left(W_m^{(i) I+II}\right) = \sum_K \omega_K |\xi^{II}_{\rho K}\rangle\langle\xi^{II}_{\rho K}|$$

Thus
$$\Pi(\sigma') = T_r\left[T_r^{I}\left(W_m^{(i) I+II}\right) P_{|\eta^{II}_{\sigma'}\rangle}\right]$$

$$= \sum_K \omega_K |\langle\eta^{II}_{\sigma'}|\xi^{II}_{\rho K}\rangle|^2 \tag{66}$$

Both methods therefore yield identical results.

Case(b): If $\mathcal{L}$ has been measured on S_I and the value λi obtained, what is the probability of finding the value σ' for S in S_{II}? (11)

The statistical state representation $W_m^{(i) I+II}$ of S_{I+II} prior to the measurement reduces to

$$W_f^{(i) I+II} = W_f^{(i) I} \otimes W_f^{(i) II} = P_{|\varphi^{I}_{\lambda i}\rangle} \otimes P_{|\xi^{II}_{\rho i}\rangle}$$

after the $\mathcal{L}$ - measurement. (c.f. remarks following equation 54)

Therefore

$$\Pi(\sigma') = T_r\left[T_r^{I}\left(W_f^{(i) I+II}\right) P_{|\eta^{II}_{\sigma'}\rangle}\right]$$

$$= T_r\left(W_f^{(i) II} P_{|\eta^{II}_{\sigma'}\rangle}\right) = |\langle\eta^{II}_{\sigma'}|\xi^{II}_{\rho i}\rangle|^2 \tag{67}$$

Again both methods give identical results.

If $S = R$ we get equation (59) provided that $\sigma' = \rho i$

If $\sigma' \equiv \rho'$ we can also write $|\langle\xi^{II}_{\rho i}|\xi^{II}_{\rho'}\rangle|^2 = \delta_{\rho i \rho'}$

"so that a definite result is predicted" says Furry. He continues:

The possibility of such definite predictions was taken by E.P.R. as a 'criterion of the physical reality' of the observable $\mathcal{R}$; it is, par excellence, the bit of evidence which might incline one to believe Assumption A to be true. (12)

Case (c): If $\mathcal{M}$ has been measured on S_I and the value μ' obtained, what is the probability of finding the value ρi for $\mathcal{R}$ in S_{II}? [in other words we want to know $\Pi(\rho i / \mu')$.] (13)

Method A: As in Case(a), we are in view of $\Pi(\mu')$ only concerned with S_I. Therefore the statistical information of S_I, i.e. $W_m^{(1)I}$, is given because of equation (54) as a mixture of the states $|\varphi^{I}_{\lambda K}\rangle$. Thus

$$\Pi(\mu') = T_r\left[T_r^{II}\left(W_m^{(1)I+II}\right)P_{|\psi^I_{\mu'}\rangle}\right] = \sum_K \omega_K |\langle \psi^I_{\mu'} | \varphi^I_{\lambda K}\rangle|^2$$

But $\Pi(\mu' \wedge \rho i)$ is the probability of μ' and S_{II} is in the particular state $|\varphi^{II}_{\rho i}\rangle$; that is,

$$\Pi(\mu' \wedge \rho i) = \sum_K \omega_K |\langle \psi^I_{\mu'} | \varphi^I_{\lambda K}\rangle|^2 \delta_{iK}$$

Since $\Pi(\mu' \wedge \rho i) = \Pi(\rho i | \mu')[\Pi(\mu')]^{-1}$. we get

$$\Pi(\rho i | \mu') = \frac{\sum_K \omega_K |\langle \psi^I_{\mu'} | \varphi^I_{\lambda K}\rangle|^2 \delta_{iK}}{\sum_K \omega_K |\langle \psi^I_{\mu'} | \varphi^I_{\lambda K}\rangle|^2}$$

$$\Pi(\rho i | \mu') = \frac{\omega_i |\langle \psi^I_{\mu'} | \varphi^I_{\lambda i}\rangle|^2}{\sum_K \omega_K |\langle \psi^I_{\mu'} | \varphi^I_{\lambda K}\rangle|^2} \tag{68}$$

Method B: If $\mathcal{M}$ has been measured on S_I and the value μ' obtained, S_{II} is in the pure state with state vector given--apart from normalization--by

$$|\zeta_{\mu'}^{II}\rangle = \sum_K (\omega_K)^{1/2} \langle \psi_{\mu'}^I | \varphi_{\lambda K}^I \rangle | \xi_{\rho K}^{II} \rangle \tag{69}$$

Then

$$\Pi(\rho_i|\mu') = \frac{|\langle \xi_{\rho i}^{II} | \zeta_{\mu'}^{II} \rangle|^2}{\langle \zeta_{\mu'}^{II} | \zeta_{\mu'}^{II} \rangle} \tag{70}$$

$$= \frac{|\sum_K (\omega_K)^{1/2} \langle \psi_{\mu'}^I | \varphi_{\lambda K}^I \rangle \langle \xi_{\rho i}^{II} | \xi_{\rho K}^{II} \rangle|^2 \delta_{ik}}{\sum_K \omega_K |\langle \psi_{\mu'}^I | \varphi_{\lambda K}^I \rangle|^2} \tag{70a}$$

$$= \frac{\omega_i |\langle \psi_{\mu'}^I | \varphi_{\lambda i}^I \rangle|}{\sum_K \omega_K |\langle \psi_{\mu'}^I | \varphi_{\lambda K}^I \rangle|^2} \tag{70b}$$

which is the same result as that of Method A.

Case (d): If $\mathcal{M}$ has been measured on S_I and the value μ' obtained, what is the probability of finding the value σ' for S in S_{II}? (14)

<u>Method A</u>: The statistical state representation of S_{I+II} is given (because of <u>Assumption A</u>) by $W^{(i)I+II}$ i.e. equation (57) Therefore

$$\Pi(\mu') = Tr\left(W_m^{(i)I} P_{|\psi^I_{\mu'}\rangle}\right) = \sum_k \omega_k \left|\langle \psi^I_{\mu'} | \varphi^I_{\lambda k}\rangle\right|^2$$

[We are, as in Case(a) and Case(b) only concerned with S_I when we calculate $\Pi(\mu')$] Then

$$\Pi(\mu' \wedge \sigma') = \omega_k \left|\langle \psi^I_{\mu'} | \varphi^I_{\lambda k}\rangle\right|^2 \left|\langle \eta^{II}_{\sigma'} | \xi^{II}_{\rho k}\rangle\right|^2$$

<u>when</u> S_{II} is in the particular state $|\xi_{\rho k}^{II}\rangle$. Therefore for all k we get

$$\Pi(\sigma' | \mu') = \frac{\Pi(\mu' \wedge \sigma')}{\Pi(\mu')}$$

$$= \frac{\sum_k \omega_k \left|\langle \psi^I_{\mu'} | \varphi^I_{\lambda k}\rangle\right|^2 \left|\langle \eta^{II}_{\sigma'} | \xi^{II}_{\rho k}\rangle\right|^2}{\sum_k \omega_k \left|\langle \psi^I_{\mu'} | \varphi^I_{\lambda k}\rangle\right|^2}. \tag{71}$$

<u>Method B</u>: <u>After</u> a measurement on S_I has given the value μ' for all $\mathcal{M}$, S_{II} is in the <u>pure state</u> with state vector given--apart from normalization--by equation (69) i.e.

$$|\xi^{II}_{\mu'}\rangle = \sum_k (\omega_k)^{1/2} \langle \psi^I_{\mu'} | \varphi^I_{\lambda k}\rangle |\xi^{II}_{\rho k}\rangle$$

Upon normalizing we get

$$\left| \xi_{\mu'}^{II} \right\rangle = \frac{\sum_K (\omega)^{1/2} \left\langle \psi_{\mu'}^{I} \middle| \varphi_{\lambda K}^{I} \right\rangle \left| \xi_{\rho K}^{II} \right\rangle}{\sum_K (\omega_K)^{1/2} \left\langle \psi_{\mu'}^{I} \middle| \varphi_{\lambda K}^{I} \right\rangle}$$

Then (c.f. equation 65 and discussion of Method B p.29)

$$\Pi(\sigma'|\mu') = \frac{\left| \left\langle \xi_{\mu'}^{II} \middle| \eta_{\sigma'}^{II} \right|^2}{\left\langle \xi_{\mu'}^{II} \middle| \xi_{\mu'}^{II} \right\rangle}$$

$$= \frac{\left| \sum_K (\omega_K)^{1/2} \left\langle \psi_{\mu'}^{I} \middle| \varphi_{\lambda K}^{I} \right\rangle \left\langle \eta_{\sigma'}^{II} \middle| \xi_{\rho K}^{II} \right\rangle \right|^2}{\sum_K \omega_K \left| \left\langle \psi_{\mu'}^{I} \middle| \varphi_{\lambda K}^{I} \right\rangle \right|^2} \quad (72)$$

Furry concludes:

> The difference between [equation (71)] and [equation(72)] comes from the well-known phenomenon of "interference" between probability amplitudes. The absence of such an effect in [Cases (a), (b), and (c) where either one or both of $\mathcal{M}$ or S is equal to $\mathcal{L}$ or $\mathcal{R}$ respectively] . . . is usually stressed in discussions of the theory, since it shows plainly the effect which the mere attaching of an instrument must in general have on the behaviour of a system. Since case (d) [neither $\mathcal{M}$ nor S is respectively the same as $\mathcal{L}$ or $\mathcal{R}$] is not mentioned, it is possible for a reader to form the impression that the theory is consistent with Assumption A.
>
> The formal discrepancy between [equation(71)] and [equation(72)] is a consequence of the fact that . . . after a measurement of $\mathcal{M}$ on system I has been made

system II is in a pure state, which is in general not one of the $\xi_{\rho k}$. Now no possible manipulation of the ω_k will produce from the statistics of the mixture those of any pure state other than one of $\xi_{\rho k}$. Thus not only is Method A inconsistent with Method B, but also there is no conceivable modification of method A which could produce consistency between Assumption A and Method B. (15)

CHAPTER III

EMPIRICAL AND LOGICAL RELATIONSHIP
BETWEEN E.P.R.'s CRITERION OF PHYSICAL REALITY
AND ASSUMPTION A

1. Introductory Remarks

In a letter to the editor, March 2, 1936, Furry clarifies and reaffirms his position in response to two papers by Erwin Schrödinger also published in 1935. (1) In particular Furry points out three important properties of Assumption A:

Ap1: The corresponding picture of the situation is in full accord with our habitual attitudes, and is the one we use in ordinary practice.

Ap2: The predictions derived from Assumption A for the sorts of cases which actually occur, cases (a), (b), (c), agree exactly with those of Quantum Mechanics.

Ap3: In more general cases, realizable in principle according to the postulates of the theory, case (d) , there is flat contradiction between the formulas given by Assumption A and those of Quantum Mechanics. Therefore Assumption A is actually untrue. (2)

Notice that the conclusion of Ap3 is actually a declaration of faith in quantum theory, since in Furry's time no experiment had shown that the postulates of quantum theory are valid in an E.P.R. type situation where the systems are separated by a large distance. Therefore, the conclusion of Ap3, that Assumption A is actually untrue is based on the faith that the postulates of quantum theory hold in an E.P.R. type situation. More specifically, Furry assumes that the formulation of the many-body problem in quantum theory does not break down in a fundamental way when

S_I and S_{II} are separated by macroscopic distances.

Realizing this, the formal discrepancy between equations II(71) and II(72) is just that, a formal discrepancy, and Furry's demonstration of the same merely shows that Assumption A stands in conflict with Method B. Method B , however, is based on a quantum mechanical formalism for the many-body problem which had in Furry's time only been experimentally verified for atomic distances.

Furry's faith in quantum theory and the unilateral correctness of its postulates was not shared by Einstein who, on the contrary, proposed "that the current formulation of the many-body problem in quantum mechanics may break down when particles are far enough apart." (3) Einstein's faith in turn rests on a definite conception of physical reality. He says:

> But on one supposition we should, in my opinion, absolutely hold fast: the real factual situation of the system S_2 is independent of what is done with the system S_1, which is spatially separated from the former . . . the real situation of S_2 must be independent of what happens to S_1 . . . (One can escape from this conclusion only by either assuming that the measurement of S_1 (telepathically) changes the real situation of S_2 or by denying independent real situations as such to things which are spatially separated from each other. Both alternatives appear to me entirely unacceptable.) (4)

However, according to Method B the state assigned to S_{II} is dependent on the physical history of S_I. Why?--Because quantum mechanical formalism was applied, and was assumed to be true for an E.P.R. type situation, and case (d) was therefore assumed to be realizable in principle at least.

The formal discrepancy between equations II(71) and II(72) arises because of two incompatible assumptions. This is what Furry has shown formally most eloquently for case (d).

Hooker puts it this way:

> What Furry's work does is draw attention to the fundamental fact that <u>the forms appropriate to our description of the world are intimately related to the conception we have of the reality to be described.</u> (The converse is also true; the conceptions we form of reality are intimately related to the descriptive forms which we find to be experimentally successful).(5)

In a recent paper Bohm and Aharonov (6) claim that an experiment of Wu and Shaknov (7) (on the correlation of polarization of annihilation photons) can be considered as empirical evidence against <u>Assumption A</u>. Its empirical results, so they claim, show that the many-body Schrodinger equation does not break down in an E.P.R. type situation where two once interacting systems are separated such that by hypothesis, any interaction between them becomes negligible.

According to the above mentioned authors, this does not resolve the so called E.P.R. paradox, but rather strengthens the "paradoxicality" of a now <u>physically</u> realized E.P.R. <u>Gedanken-experiment</u>. The Wu-Shaknov experiment prevents us from escaping the E.P.R. paradox. It eliminates the possibility which Einstein had hoped for, namely to treat two non-interacting, spatially separated systems classically, through the use of statistical mixtures in the classical sense. It prevents us therefore from escaping the non-local features of quantum theory implied by the Superposition Principle even in a case where the separation between S_I and S_{II} is macroscopically arbitrarily large.

The above remarks will have to be examined more closely. We shall do this by giving an account of Bohm's reformulation of the E.P.R. paradox and restate <u>Assumption A</u> in terms of the same.

2. Bohm's Reformulation of the E.P.R. Paradox

Bohm's reformulation avoids the involvement of continuous dynamical conjugate variables such as position and momentum. He considers a situation in which only discrete physical quantities such as spin eigenvalues are involved, a situation more easily manageable experimentally.

If we consider a diatomic molecule of total spin zero, each of whose atoms has spin one-half, then the wave function of the system is given by

$$\psi_0 = \frac{1}{\sqrt{2}}\left[\psi_+(I)\,\psi_-(II) - \psi_-(I)\,\psi_+(II)\right] \qquad (73)$$

where $\psi_+(I)$ and $\psi_-(II)$ are wave functions representing the atomic states of particle A having spin $+\frac{\hbar}{2}$ and particle B having spin $-\frac{\hbar}{2}$ respectively.

We now suppose that the diatomic molucule is separated by some force that breaks the relatively weak bond of the molecule, but in such a way that the total spin is not disturbed, that is, no torque is exerted on either atom. After particles A and B have separated enough and interaction between them has ceased, we can measure any desired spin component, say of particle A, by detecting the motion of particle A in an inhomogeneous magnetic field. To be more specific, the measurement of a spin component may be performed by observing whether the particle emerges with an upward or downward deflection from a Stern-Gerlach experiment.

Since on account of equation (73) the total spin will remain zero after separation of particles A and B, [i.e., e.g. $S_z(A) = -S_z(B)$; no torque was exerted on either atom] the two systems are correlated for all times in such a way that whenever we

take a measurement of any of the possible spin components of A, say z, we can immediately conclude that B also has a z - spin component that is equal but opposite to that of A.

However, the Indeterminacy Principle does not allow us to measure more than one spin component of either system at any given time. The components of the spin in another direction, once we have chosen the coordinate on which the net spin is to be measured, must remain undetermined. Suppose the z - component of a particle is definite; then the x and y components are indefinite, and we may regard them more or less to be in a kind of "random fluctuation."

The possibility of being able to determine only one component of a physical system at any given time is foreign to a classical conception of physical reality. From a classical point of view, <u>each</u> of the spin-components of particle A would enjoy the ontological status of a well defined, existing physical quantity. Particle B likewise would <u>have</u> all components simultaneously but equal and opposite to those of particle A at any given time, provided conditions of symmetry are conserved throughout.

But the crucial issue of the spin example is, in spite of the restrictions of the Indeterminacy Principle, that no matter what spin component of particla A we shoose to measure, the spin component of particle B will always have a definite, equal and opposite spin component to the measured spin component of particle A.

From equation (73) we can also write:

$$\psi_0 = \frac{1}{\sqrt{2}}\left[\phi_{+-} - \phi_{-+}\right] = \frac{1}{\sqrt{2}}\begin{vmatrix} |\phi_+^{I}\rangle & |\phi_+^{II}\rangle \\ |\phi_-^{I}\rangle & |\phi_-^{II}\rangle \end{vmatrix}$$

where $\phi_{+-} \equiv |\phi_+^I\rangle \otimes |\phi_-^{II}\rangle = \psi_+(I)\,\psi_-(II)$ and

$$\phi_{-+} \equiv |\phi_-^I\rangle \otimes |\phi_+^{II}\rangle = \psi_-(I)\,\psi_+(II)$$

ψ_0 is an interference property of both ϕ_{+-} and ϕ_{-+}. The only states in which each particle has a definite spin opposite to that of the other particle, are represented either by ϕ_{+-} or ϕ_{-+} separately. That is, after a measurement of a certain spin component, ψ_0 reduces to either ϕ_{+-} or ϕ_{-+}, and the total angular momentum expressed by ψ_0, i.e. equation (73), before the measurement is now indefinite. On the other hand, before a measurement, the state of the system is given by ψ_0 and the angular momentum is definite but neither particle can correctly be regarded as having a definite value of its own spin, for if it did, there could be no interference between ϕ_{+-} and ϕ_{-+} and it is just this interference which is required to produce a definite total angular momentum. (8)

Bohm and Aharonov point out (9) that the usual interpretation of the Indeterminacy Principle, as a consequence of the essentially uncontrollably interaction between object and measuring device makes perfect sense when measurements are performed on a single system. In such a case one spin component becomes definite whereas the others become indefinite. Before a measurement all the spin components are in some sort of fluctuation and any one component's definiteness is an "incompatible potentiality" that may be actualized through a suitably oriented measuring apparatus.

But according to Bohm and Aharonov, (10) this interpretation of the Indeterminacy Principle falls down for an E.P.R. type situation

which requires:

E(a): That the two particles do not interact significantly

E(b): Consequently, the measuring device interacts with only one system at a time.

If both E(a) and E(b) are satisfied, then the usual interpretation of the Indeterminacy Principle is unsatisfactory for two reasons:

(1) It does not explain why particle B (which does not interact with A or with the measuring apparatus) realizes its potentiality for a definite spin in precisely the direction as that of A.

(2) It cannot explain the fluctuation of the other two components of the particle B as the result of disturbance due to the measuring apparatus. (11)

To explain the above situation by postulating some sort of hidden interaction potential between particles A and B is unsatisfactory for two reasons. Firstly, an explanation in terms of a hidden interaction potential between particles A and B and the measuring device is outside the scope of current quantum theory. Secondly, such a hidden interaction potential:

> . . . would have to be instantaneous, because the orientation of the measuring apparatus could very quickly be changed [ψ_0 is invariant under spatial rotation, c.f.p. 39] and the spin of B would have to respond immediately to the change. Such an immediate interaction between distant systems would not in general be consistent with the theory of relativity.
> This result constitutes the essence of the paradox of Einstein, Podolsky and Rosen. (12)

Digression: Bohm's and Aharonov's second objection to the postulate of a hidden interaction potential, seems somewhat irrelevant since, because of equation (73) we are dealing from the very outset within

the framework of non-relativistic quantum theory. Their objection therefore that the postulate of a hidden interaction potential is unsatisfactory because it conflicts with relativity is misleading and without force.

Moreover, since the E.P.R. paradox and Bohm's reformulation of the same, is an appeal to what we consider physically plausible or reasonable, the word "paradox" as used in the above context ought to be understood in terms of its original meaning rather than in terms of the more precise meaning given to it in logic. In its original sense, the word "paradox" implies an embracing of conflicting ideas. Its Greek root (p a r a - d o x o n) denotes something unexpected, something contrary to common sense. In general, a paradox is any conclusion that goes against generally accepted opinion but has an argument to sustain it. Quine remarks:

> The argument that sustains a paradox may expose the absurdity of a buried premise or of some preconception previously reckoned as central to physical theory, to mathematics, or to the thinking process. Catastrophy may lurk, therefore, in the most innocent-seeming paradox. More than once in history the discovery of paradox has been the occasion for major reconstruction at the foundations of thought. (13)

However, one may object from the very outset and argue that there is nothing paradoxical at all about the E.P.R. type situation or Bohm's reformulation of the same. One is of course always free to argue that way, provided one has good reasons to do so. A case in point, for example, would be to reject Assumption A, and with it its property Apl (c.f.p. 36). But as will be seen, the rejection of Assumption A adds to the "paradoxicality" of the physical phenomena in question, precisely because it appears that we must reject Assumption A and therefore Apl, that is, our habitual attitudes which are based on a classical conception of physical reality.

The rejection of Apl, which necessitates the adoption of some new epistemological criterion for that which can be called physically real is, for some physicists, not burdensome at all. They simply give up their "habitual attitudes" when it comes to certain issues in microphysics and tend to ignore those scientists who do have difficulties in rejecting Apl. To simply eliminate Apl as a matter of convenience, is unsatisfactory both from a scientific as well as from a philosophical point of view, unless the rejection of Apl is supoorted by physical and/or philosophical arguments, as for example in the case of Bohr.

One may further attempt to avoid the paradox by arguing that we are dealing here only with harmless correlations. This is, as Hooker points out "an admittedly attractive position . . . but . . . this view is factually as well as conceptually inadequate for the situation." (14) He continues:

> The really key consideration here is not what a catalogue of experimental results, by itself, can tell us, but what the theory tells us--and the theory tells us that the quantum states change depending on what is being measured. What should be decisive here, even for tough-minded empiricist physicists, is that the differing conceptions of the physical reality in these situations lead to objectively different statistical results. (15)

In a footnote he enlarges his remarks by saying:

> Physicists, especially, are apt to draw attention to the allegedly unproblematic probability relations involved in just the experimental results (of the common cases only . . .) [Hooker is referring here to Furry's cases (a),(b),(c)] because they tend to share a positivist bias towards downgrading the significance of theory as a serious element in physical understanding; only experimental results are supposed to count. Of course, if one practices their approach severely enough, any theoretical problem whatever can be solved. But this is essentially an uninteresting and implausible

> position to adopt . . . I am therefore not tempted, for example, to take very seriously the confused and extremely positivistic criticisms by Beitenberger [On the So-Called Paradox of Einstein, Podolsky and Rosen IL NUOVO CIMENTO, Vol. XXXVII, No. 1] of those who take the quantum paradoxes seriously. (16)

Furthermore, one may also attempt to remove the paradox on purely formal grounds, without however being overly concerned about what happens physically, and how it is possible that physical phenomena correspond to quantum mechanical formalism and visa versa. For example, Hooker has this to say about Jauch's formalistic approach to the spin-problem:

> Jauch's theory further tells us that, for any given direction, after a measurement has been made the final state of the particles (for spins) of photons (for polarizations) is a mixture representing one of the possible outcomes (spin $+\frac{h}{2}$, $-\frac{h}{2}$ and so on.). All of this is fine and beautifully consistent. What it does not even begin to explain, but merely takes for granted, is how on earth changing the direction in which the spin (or polarization) measuring instrument is to act on one of the component states succeeds in bringing the spin (or polarization) state of the other component into agreement with it, when the second component is completely physically isolated (interaction potential zero) from the former component. One wants, not a formal explanation of the situation, but a physical account of how it comes about. One wants the situation to be made physically plausible. (17)

<u>End of Digression</u>

Since, however, the conditions E(a) and E(b) of an E.P.R. type situation have not been realized experimentally, in other words the many-body equations of Schrödinger and Dirac have not been tested under conditions E(a) and E(b), we may still conclude that there is no empirical evidence that the physical phenomena of an E.P.R. type situation satisfying conditions E(a) and E(b) will really turn out to be paradoxical.

If it should turn out experimentally that the many-body Schrödinger and Dirac equations will break down under conditions E(a) and E(b), then the E.P.R. Thought-experiment is physically realized but in such a way as to render Assumption A as physically valid. Then the probability calculations of certain spin values could legitimately be carried out according to classical probability theory, since then each spin state belongs to an ensemble of individual spin states, where each spin state is independent from any other spin state, and thus constitutes a well defined state in the classical sense, endowed with real and well defined properties. That is, the physical phenomena of an E.P.R. type situation would then not be paradoxical at all.

If, on the other hand, it should turn out experimentally that the Schrödinger and Dirac equations of the many-body problem do not break down under conditions E(a) and E(b), then Assumption A would not only be formally inconsistent with Furry's Method B, but would also have to be rejected on physical grounds and would break down for Furry's case (d) both formally as well as empirically.

3. Assumption A Restated in Terms of the Spin Problem

Assumption A in terms of the spin problem can be stated as follows: After the state of spin zero decomposes, the wave function ψ_0 for the total system is eventually no longer given by

$$\psi_0 = \frac{1}{\sqrt{2}}\left[\psi_+(I)\,\psi_-(II) - \psi_-(I)\,\psi_+(II)\right] = \frac{1}{\sqrt{2}}\left[\phi_{+-} - \phi_{-+}\right]$$

"which implies the puzzling correlations of the spin of the two atoms", but we assume that in each individual case, "the spin of each atom becomes definite in some direction while that of the other atom is opposite." (18) Then the wave function will be either the product ϕ_{+-} or ϕ_{-+}. That is, (using in part Bohm and

Aharonov's notation)

$$\psi = \phi_{+-} = \psi_{+\theta\varphi}(I)\,\psi_{-\theta\varphi}(II)$$

or (74)

$$\psi = \phi_{-+} = \psi_{-\theta\varphi}(I)\,\psi_{+\theta\varphi}(II)$$

where

$\psi_{+\theta\varphi}(I)$ for example is a wave function of particle A whose spin is positive in the direction given by θ and φ .

According to Assumption A each particle goes into some unpredictable, but classically well defined state. The "fluctuations" of the other two spin components of particle A are now uncorrelated to those of particle B.

Although Assumption A entails the breakdown of the conservation of the total momentum for a particular instance, it is possible to retain spherical symmetry and angular momentum conservation in the statistical sense if we require uniform probability for any direction of θ and φ in a large number of similar cases.

However, since on the basis of Assumption A there is no guarantee that we will always find $S_z(A) + S_z(B) = 0$ for every single case, we can no longer speak of a precise correlation between arbitrary spin components of particles A and B for every case. Therefore "our decision to choose a certain direction for measuring the spin of particle A will have no influence whatever on the state of particle B" (19), since ψ is just the product ϕ_{+-} or ϕ_{-+} c.f.equation (74) . This means with respect to Furry's case (d) that without having made a measurement on either system, we know only that the probabilities of the different transitions of S_I and S_{II} are respectively w_k such that if S_I is in the spin state S_{z_i} (A) then S_{II} is in the spin state S_{δ_i}(B). Therefore all further probability calculations can be performed by methods of ordinary probability calculus.

4. Logical Relation Between Assumption A and E.P.R.'s Criterion of Physical Reality

What is assumed here, according to Furry is "the independent existence of two entities" (20), (i) the state of S_{II} and (ii) one's knowledge of S_{II}, with only one's knowledge of S_{II} (But not the state of S_{II} itself) being affected by measurements made on S_I.

Thus classical probability theory applies and we do not have to reckon with interference terms of probability amplitudes, escaping therefore the non-local features of quantum theory implied by the Superposition Principle.

But what Furry shows is, as Hooker puts it, that the:

> . . . reduction of the wave packet on measurement is a real, physical, and significant change because the physical properties of the composite system are different from those of the corresponding mixture; therefore the system, and a fortiori its components, cannot be regarded as being in a mixture both before and after a measurement . . . (21)

Now if it were the case that Assumption A correctly applies to an E.P.R. type situation, then the E.P.R. paradox would no longer exist, since we would than be faced with a classical situation in which two independent non-interacting systems with well defined properties are correlated--but in such a way that only our knowledge of S_{II} would be affected by measurements made on S_I, and not the state of S_{II} itself.

Clearly the decision as to whether or not we are nonetheless faced with a paradox depends upon, whether or not Assumption A is physically justified. Bohm and Aharonov, as was mentioned in the beginning, claim that the Wu-Schaknov experiment can be considered as empirical evidence against Assumption A, and constitutes empirical proof that the paradoxical aspects of quantum theory described by

E.P.R. "represent real properties of matter." They show that the "Wu Shaknov experiment on the polarization of the annihilation radiation of positronium provides an experimental confirmation of the features of quantum mechanisms which are the basis of the E.P.R. paradox." (22)

That is, the Wu-Shaknov experiment shows that Assumption A, namely that the many-body Schrödinger and Dirac equations break down in a fundamental way at macroscopic distances, is false. They state:

> . . . we calculated the results of the Wu-Shaknov experiment according to the Furry hypotheses Assumption A with all possible assumptions concerning the polarization states which the system separates, and we have shown that they are all inconsistent with the results of the experiment (which were already known to agree within experimental error with the predictions of the quantum theory.) (23)

Thus $\psi_0 = \frac{1}{\sqrt{2}}\left[\psi_+(I)\,\psi_-(II) - \psi_-(I)\,\psi_+(II)\right]$

does not breakdown for macroscopic distances, the unique correlation remains, and so does the "paradoxical behaviour of matter" in a now physically realized E.P.R. type situation and Furry's Method B is the correct way to calculate the probabilities for case (d).

The Wu-Shaknov experiment has added to the paradoxicality of the E.P.R. Gedanken-experiment in two ways: It is a physical experiment corresponding to the E.P.R. thought-experiment, and it shows that Assumption A must be rejected also on physical grounds.

Two questions arise immediately: Firstly, is the Wu-Ahaknov experiment empirically conclusive? Secondly, assuming it is, does the rejection of Assumption A necessitate us also to reject E.P.R.'s criterion of physical reality?

In response to the first question, Hooker remarks that Bohm and Aharonov:

> . . . used, for example, the quantum mechanical scattering formula to obtain the scattering probabilities for these definite-state photons . . . The Wu-Shaknov experiment, therefore, only falsifies an E.P.R. type reality assumption against the background of the remainder of quantum mechanical methods . . . It has nothing to say concerning the possibilities when quantum theoretical methods are abandoned in a more radical way. Thus it would only bear upon a hidden variable theory that not only determined the photon polarizations, but also retained the scattering formula. But such a theory is implausibly close to quantum theory anyway. The next weakest class of theories is that considered by Bell, who postulates merely that in the E.P.R. type cases, the theory is local . . . In this case there is no reason to retain the quantum methods of calculation and in fact Kasday has succeeded in constructing a hidden variable model of the Wu-Shaknov situation which abandons the quantum mechanical scattering formula and which yields the correct experimental result. (24)

Yet, in physical situations, e.g. Furry's case (d), in which discrepancies arise between quantum mechanical methods and classical methods, experimental results favour quantum mechanical methods over methods based on a classical conception of physical reality.

We must be willing, therefore, to give an affirmative answer to the first question, keeping in mind, however, as was pointed out by Hooker, the relative strength and weakness of the Wu-Shaknov experiment "in order to understand its relations to hidden variable theories and other experimental possibilities." (25) But this makes the second question even more crucial.

Whether or not the rejection of <u>Assumption A</u> entails the rejection of E.P.R.'s criterion of physical reality depends upon whether or not E.P.R.'s criterion of physical reality entails <u>Assumption A</u>.

A closer examination of E.P.R.'s criterion of physical reality examined within the context of the special E.P.R. type situation reveals that it does indeed, together with some additional minor assumptions, entail <u>Assumption A</u>, i.e. (Ai), (Aj), (Ak) (c.f.p.28)

We restate E.P.R.'s criterion of physical reality:

C_b: If, without in any way disturbing a system, we can predict with certainty (i.e. with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity. (26)

Applying C_b to the special E.P.R. type situation (c.f. Chapter I p. 8) we reason as follows: S_I and S_{II} interact from $t = 0$ to $t = T$ but separate in such a way that at $t > T$, $q = Q$ relative to an arbitrary position coordinate x_o, and $p = -P$. We can then predict with certainty the value Q of S_{II} and visa versa. Similarly, a measurement of p on S_I at $t > T$ allows us to predict with certainty -P of S_{II} and vice versa.

We are able to predict with certainty (i.e. with probability equal to one) the values of the respective physical quantities without disturbing the system to which they belong, <u>because it is assumed that after S_I and S_{II} have separated no physical interaction exists between them at $t > T$.</u> Furthermore, as is evident, the correlation between S_I and S_{II} is one of symmetry. These conditions are satisfied for all time $t > T$.

Since for $t > T$ S_I and S_{II} are in definite though unknown states, "we cannot, however, calculate the state in which either one of the two systems is left after the interaction. This, according to quantum mechanics, can be done only with the help of further measurements. . ." (27)

From this it follows that C_b entails (Ai), i.e. each system made a transition to a definite though unknown state.

Secondly, the state in which either one of the two systems is left after the interaction can be discovered according to quantum theory only by further measurements. This entails (Aj).

Thirdly, the requirement of no interaction between S_I and S_{II}, and the fact that S_I and S_{II} are in definite though unknown states, allows us, when not knowing which states they are in, to make use of ordinary probability calculus for all necessary calculations. Therefore (Ak) is implied, if we grant the assumption that only classical probability theory is appropriate here.

Thus it has been shown that <u>Assumption A</u> is entailed by C_b for the special E.P.R. type situation, for which, according to Furry, <u>Assumption A</u> is a perfectly good working hypothesis.

However, Schrödinger rejected <u>Assumption A</u> but accepted E.P.R.'s criterion of physical reality. But Furry correctly points out about Schrödinger:

> Since he agrees with the underlying assumption of E.P.R., he is careful to introduce no <u>a priori</u> "doubtful" element into his thesis. Thus he rejects . . . Assumption A, and ends with taking as his criterion of "reality" just that of E.P.R. (28)

Furry quotes Schrödinger as saying:

> The "real" properties of the "free" system are the values of those observables whose values could be predicted "without in any way disturbing the system." (29)

Furry continues:

> On this basis one can give interesting considerations only about certain degenerate situations, such as that chosen as an example by E.P.R. (30)

Furry now makes the following claims regarding C_b:

C_b(p1): It is in the opinion of E.P.R. and of Schrödinger indubitably true a priori.

C_b(p2): None of its assertions--it is innocent of actual predictions--ever comes into direct contradiction with the results of quantum mechanics; hence it cannot be either proved of disproved objectively. (31)

But we saw that Assumption A is inconsistent both formally as well as empirically for Furry's case (d) and must therefore be rejected if quantum theory is correct. And since it was shown that C_b entails Assumption A, it follows, that if Assumption A is untrue and must be rejected if quantum theory is correct, then E.P.R.'s criterion of physical reality (C_b) must also be rejected.

Therefore, contrary to Furry's claim C_b(p2), C_b does come into conflict with quantum theory. E.P.R.'s criterion of physical reality is therefore not innocent of actual predictions. As Hooker puts it:

> . . . A2 [C_b], far from being "innocent of actual predictions, far from not conflicting with quantum theory, has all of the consequences of A [Assumption A] (32)

Had Furry realized this (c.f. quotation from Hooker p. 38 above) he could have argued his case even more strongly.

But we must be cautious and take notice of the fact that E.P.R. never questioned the correctness of quantum theory. As will be remembered, a successful theory is both correct and complete. "Correctness of a theory is judged by the degree of agreement between the conclusion of the theory and human experience, which in physics takes the form of experiments and measurements." (33)

It was on the basis of experimental results and therefore on the basis of the correctness of quantum theory and on that basis alone, that Assumption A and consequently E.P.R.'s criterion of physical reality was abandoned.

However, the correctness of a theory does not necessarily entail its completeness. This still therefore leaves the question open as to whether or not quantum theory is complete, that is, whether or not, as is usually assumed, the wave function does contain a complete description of "physical reality" of the system in the state to which it corresponds.

Digression: It may, of course, be questioned whether or not E.P.R.'s distinction between a correct and a complete theory is legitimate or even useful. But we do not think it to be useful to enter into a discussion of this subtle point at this stage of the argument. It must be remembered that the heart of the E.P.R. objection, its subject matter, is Physics, and any discussion of the cognitive aspects of scientific theories that enter into E.P.R.'s arguments are important only insofar as they help to clarify what is physically reasonable and plausible.
End of Digression

We argued that at this point it is still undecided whether or not the wave function does contain a complete description of physical reality. We may therefore raise the following question: If E.P.R.'s criterion of physical reality is to be rejected, with what other kind of criteria of physical reality is it to be replaced? Quantum reality? Let us suppose this is the case and for want of a better name replace C_b by "Quantum reality" (i.e. $C_b{}^*$) where "quantum reality" denotes a criterion for the physical reality of a physical quantity which is different from C_b and more satisfactory, at least in the sense that it does not entail Assumption A, such that

$$\neg \left[C_b{}^* \supset (\text{Assumption A}) \right]$$

and therefore $\left[C_b{}^* \wedge \neg \left(\underline{\text{Assumption A}}\right) \right]$ is true.

Then E.P.R.'s necessary requirement for a complete theory, and specifically now for quantum theory, would read as follows:

$C_a{}^*$: Every element of "quantum reality" (i.e. $C_b{}^*$) must have a counterpart in quantum theory.

Now depending on what this new criterion of physical reality, that is $C_b{}^*$, turns out to be, it can indeed be shown that quantum theory is complete, or as complete as it can ever be, and as it turns out, even must be.

In fact, E.P.R. were probably well aware of this when they made the following statements:

> We are thus forced to conclude that the quantum-mechanical description of physical reality given by wave functions is not complete.
> One could object to this conclusion on the grounds that our criterion of physical reality is not sufficiently restrictive. Indeed, one would not arrive at our conclusion if one insisted that two or more physical quantities can be regarded as simultaneous elements of reality only when they can be simultaneously predicted. On this point of view, since either one or the other, but not both simultaneously, of the quantities P and Q can be predicted, they are not simultaneously real. This makes the reality of P and Q depend upon the process of measurement carried out on the first system, which does not disturb the second system in any way. No reasonable definition of physical reality could be expected to permit this. (34)

To further examine the epistemological issues involved here, we shall in what follows focus our attention on two papers by Schrödinger which were also published in 1935. (35)

CHAPTER IV

AN EXAMINATION OF SOME EPISTEMOLOGICAL ISSUES IN QUANTUM THEORY

1. Introductory Remarks

We saw in Chapter III that E.P.R.'s criterion of physical reality, C_b, must be abandoned if quantum theory is true. It was illustrated in Chapter II and III that for certain quantum mechanical situations, formal as well as empirical, considerations strongly favour the rejection of a classical conception of physical reality that lies at the basis of Assumption A. To what extent is this rejection justified and what are some of its theoretical and epistemological consequences?

2. Classical Concept of State and Model versus Quantum Mechanical Concept of State of Physical Systems

We start with Furry's assertion that:

> . . . it is not correct to assume that the only physically significant relations are those which are directly obvious from a classical model, and it is on the basis of such a classical model that Assumption A rests. (1)

What Furry essentially expresses is the usual interpretation of quantum theory. This interpretation implies that we must renounce the possibility of describing a microscopic system in terms of a precisely defined conceptual model in the classical sense. This interpretation therefore assumes that it is not possible to give an objective and precisely definable description of physical reality at the quantum level of accuracy.

What we shall generally mean by the expression "model" or "conceptual model" here, and in what follows, is the description or representation of our simplified "picture" of a physical object or a physical system.

It will be remembered that E.P.R. postulate Objective Reality as independent from any theory and the physical concepts with which the theory operates, and that these physical concepts of a theory are intended to correspond to Objective Reality, and that it is by means of these physical concepts that we "picture" Objective Reality to ourselves.

On the basis of the above postulate, a distinction can be drawn between "Laws" that constitute Objective Reality governing the immanent pattern of Nature independent of our knowledge and the conglomeration of theories, scientific laws, ("our laws"), and physical concepts by means of which we "picture" Objective Reality to ourselves. According to Einstein, (2) progress in science is not merely a passive accumulation of data, but scientific progress rests essentially on the continuous attempt to reconstruct a more and more perfect "picture" of Objective Reality by means of physical concepts and theories.

Therefore, scientific laws, concepts, hypotheses, and conceptual models of physical reality apply, if at all, only approximately to the immanent "Laws" of Objective Reality, because scientific discovery is essentially a reconstruction (". . . operation with concepts and creation and application of definite functional association of sense experience to the concepts . . .") (3) and not merely a passive reflection of physical phenomena or sense experience. We construct models of physical reality, set forth hypotheses and employ abstract mathematical formalism.

The particular concepts and conceptual models to be used depend of course on the kind of physical situation to be described. Moreover, we must always be prepared to modify or reject a conceptual

model if, for example, through better measurement techniques it turns out that for the particular physical situation in question our model is unsatisfactory. If there is agreement with our observations and the model we employ, we say we have shosen a good model; if there is disagreement then the model or physical concepts and laws adopted are bad and must either be modified or completely rejected. We attempt therefore, to construct better and more adequate models and physical concepts that are intended to be a more appropriate reconstruction of Objective Reality.

A few examples are in order. An individual dynamical system consisting of one classical particle can be described in terms of a single precisely defined conceptual model: We consider the particle as a "mathematical point" with mass "m". To describe the position or configuration of the system, we select the smallest possible number of variables. These are called the generalized coordinates of the system. The number "f" of such coordinates is called the number of degrees of freedom of the system. A physical system, such as the Rutherford Model of the hydrogen atom can be described by some set of f coordinates q_1, q_2, . . . q_f; f = 6, and f corresponding momenta p_1; p_2, . . . p_f; f = 6, that is by a total number of 2f parameters or variables. Thus for a system of N point particles, each particle is characterized by three position coordinates such that f = 3N.

When all parameters or variables take on values we say that the model is in a definite state. Given a definite initial state, we are then able to determine how the state of the model will change with time, provided the constants of a model (which in part determine its description but not its state) such as mass and charge are specified.

Furthermore, since there is no uniqueness in the choice of generalized coordinates, the state of a model may also be determined in terms of constants of motion , such as energy, the

three momentum components of the center of mass, and the kinetic energy of the center of mass. But the total number of parameters (i.e. 2f) will always be the same. Thus for the model of the hydrogen atom, 2f = 2(3N) = 12

This classical concept of the "state" of a conceptual model that describes the physical system is lost in quantum mechanics, since in quantum mechanics at most only a "well chosen half" of the variables of a system may take on definite values, the other half being completely undetermined.

Nonetheless, all parameters, or more specifically, all canonically conjugate variables such as position and momentum, may be partially determined. The degree to which a pair of canonically conjugate variables may be determined, is expressed quantitatively through the well known Heisenberg Indeterminacy relation

$$\Delta q_i \cdot \Delta p_i \geq \frac{\hbar}{2} \qquad (75)$$

where

$$\begin{aligned} \Delta q_i &= \sqrt{\langle q_i^2 \rangle - \langle q_i \rangle^2} \\ \Delta p_i &= \sqrt{\langle p_i^2 \rangle - \langle p_i \rangle^2} \end{aligned} \qquad (76)$$

are the root-mean-square deviations of the statistical distributions of q_i and p_i respectively. This indeterminacy remains constant in time. What changes deterministically are the statistics or probabilities.

This is ensured by the fact that the state function

$$\Psi(\underset{\sim}{r}, t) = (2\pi\hbar)^{-3/2} \iiint_{-\infty}^{\infty} \phi(\underset{\sim}{p})\, e^{\frac{i}{\hbar}\left[\underset{\sim}{p} \cdot \underset{\sim}{r} - \frac{p^2 t}{2m}\right]} d^3\underset{\sim}{p} \qquad (77)$$

satisfies the differential equation

$$i\hbar \frac{\partial \Psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \Psi(x,t) = H\Psi(x,t) \quad (78)$$

which is of first order in time.

Schrödinger's equation is therefore analagous to Newton's equations of motion in classical physics, but it differs from Newtonian equations in that it determines only the probabilities of real events.

As was pointed out, the indeterminacy remains constant in time. To show this we can write in virtue of equations (76) for an Hermitian operator $\mathcal{L}$

$$\Delta \mathcal{L}_t = \sqrt{\langle \mathcal{L}^2 \rangle_t - \langle \mathcal{L} \rangle_t^2} \quad (79)$$

where

$$\langle \mathcal{L} \rangle_t = \sum_{K=1}^{\infty} |\langle u_{\Lambda K} | \Psi_t \rangle|^2 \Lambda_K \ . \quad (80)$$

Substituting equation (80) into equation (79) we get

$$\Delta \mathcal{L}_t = \left[\left(\sum_{K=1}^{\infty} |\langle u_{\Lambda K} | \Psi_t \rangle|^2 \Lambda_K^2 \right) - \left(\sum_{K=1}^{\infty} |\langle u_{\Lambda K} | \Psi_t \rangle|^2 \Lambda_K \right)^2 \right]^{1/2} \quad (81)$$

On the basis of equation (79), the time dependency of $\Delta \mathcal{L}t$ can be determined if we know the time dependency of $\langle \mathcal{L} \rangle_t$, where the time dependency of $\langle \mathcal{L} \rangle_t$ is characterized by

$$\frac{d}{dt} \langle \mathcal{L} \rangle_t \ .$$

Therefore

$$\frac{d}{dt}\langle \mathcal{L} \rangle_t = \frac{d}{dt}\langle \psi_t | \mathcal{L} \psi_t \rangle = \langle \dot{\psi}_t | \mathcal{L} \psi_t \rangle + \langle \psi_t | \mathcal{L} \dot{\psi}_t \rangle \quad (82)$$

and from equation (78)

$$\dot{\psi}_t = \frac{\partial}{\partial t}\psi_t = \frac{\partial}{\partial t}\psi(x, t) = -\frac{i}{\hbar} H \psi_t \quad . \quad (83)$$

Substituting equation (83) into equation (82) we obtain

$$\frac{d}{dt}\langle \mathcal{L} \rangle_t = \frac{i}{\hbar}\left[\langle H\psi_t | \mathcal{L} \psi_t \rangle - \langle \psi_t | \mathcal{L} H \psi_t \rangle\right] \quad (84)$$

Since H is Hermitian

$$\frac{d}{dt}\langle \mathcal{L} \rangle_t = \frac{i}{\hbar}\langle \psi_t | (H\mathcal{L} - \mathcal{L}H) | \psi_t \rangle \quad (85)$$

If $\mathcal{L}$ commutes with H then the observable represented by the Hermitian operator $\mathcal{L}$ is a constant of motion since

$\frac{d}{dt}\langle \mathcal{L} \rangle_t = 0$ and $\langle \mathcal{L} \rangle_t$ is therefore constant in time. Moreover, $\Delta \mathcal{L} t$ is time independent since both $\langle \mathcal{L}^2 \rangle_t$ and $\langle \mathcal{L} \rangle_t^2$ are constants.

It is to be noted that $\mathcal{L}$ does not itself depend explicitly on time. We are operating within the <u>Schrödinger representation</u> of quantum mechanics in which all the time dependence of observables represented by operators such as $\mathcal{L}$ is carried out by the state function. On the other hand, in the <u>Heisenberg</u>

representation, the state function is time independent and all of the time dependency is carried out by the dynamical operators themselves in terms of the equations of motion.

3. The Ontolcgical Status of Non-Commuting Variables

Although quantum mechanics rejects the classical concept of a state of a conceptual model, it finds it on the other hand appropriate that all variables of the classical model can in principle be measured. They cannot however all be precisely determined simultaneously. Thus all predictions are still predictions of variables of the classical model, such as position, momentum, energy etc.. What is non-classical is that only probabilities can be predicted. "The classical model plays in quantum mechanics a Proteus role." says Schrödinger, "each of its variables can in certain circumstances become the object of our attention and gain a certain 'reality', but never all at the same time, . . . and at most only half of a complete set of variables. . ." (4)

Schrödinger raises the following questions: What happens to the other half of the variables?

(a) Do they possess reality?

(b) Do they possess a blurred reality (Verschwommene Realität)?

(c) Or are all variables physically real except that simultaneous knowledge of them is impossible? (5)

It is very tempting to give an affirmative answer to (c) by viewing quantum mechanics as a generalization or extension of classical statistical mechanics and to assert, as does Popper, "that the observer, or better, the experimentalist, plays in quantum theory exactly the same role as in classical physics. His task is to test the theory." (6) He claims that his view is opposite to the Copenhagen interpretation of quantum mechanics which says

"that 'objective reality has evaporated', and that quantum mechanics does not represent particles, but rather our knowledge, our observations, or our consciousness of particles." (7)

Popper defends his thesis that quantum theory attempts to solve essentially statistical problems by referring to the historical development of quantum theory. He says:

> (a) It was so with Plank's problem in 1899-90 which led to his radiation formula. (b) It was so with Einstein's photon hypothesis and his derivation of Plank's formula. (c) It was so (at least in part) with Bohr's problem of 1913 which led to his theory of spectral emissions: the explanation of the Rydberg-Ritz combination principle was, clearly, a statistical problem (especially after Einstein's photon hypothesis had been proposed) . . . All this is to support my thesis that the problems of the new quantum theory were essentially of a statistical or probabilistic character. (8)

Popper further claims that "statistical questions demand essentially statistical answers" and "that statistical conclusions cannot be obtained without statistical premises. And therefore answers to statistical questions cannot be obtained without a statistical theory." (9)

In a recent paper, (10) Ballentine argues on similar lines saying that a Statistical Interpretation of quantum theory can provide a sound interpretation of the same, using a minimum of assumptions. He states that according to a Statistical Interpretation (S.I.)

S.I.: a pure state (and hence also a general state) provides a description of certain statistical properties of an ensemble of similarly prepared systems, but need not provide a complete description of an individual system. (11)

This interpretation, Ballentine claims, is upheld by Einstein, Popper and Blokhintsev, and must be distinguished from the usual concept of state based on the Orthodox Interpretation (O.I.) of

quantum theory which asserts that

O.I.: a pure state provides a complete and exhaustive description of an individual system (e.g. an electron) (12)

He then asserts that this latter interpretation is an additional and unnecessary assumption with peculiar consequences; that it is unnecessary for quantum theory and leads to serious difficulties.

Ballentine is correct in saying that the orthodox interpretation of quantum theory leads to serious difficulties, but he is rather optimistic in saying that a Statistical Interpretation can provide a sound interpretation of quantum theory using a minimum of assumptions;

We will not give a detailed account of Ballentine's Statistical Interpretation or Popper's Propensity Interpretation, but merely assert at this stage, that such interpretations do not seem to us to be far reaching enough and that the wave-particle duality phenomenon is equally difficult to come to terms with through such alternative interpretations of quantum theory.

We shall suggest later on that the attempts of Vigier, Bohm and others to explain statistical phenomena of quantum theory on the basis of "hidden variables" are optimistic as well and not without controversy, to say the least, but if successful, are more exciting and more likely to solve the difficulties that beset present quantum theory. Hidden variable theories are not mere alternative interpretations of quantum theory but are what Bohm calls "deeper theories" which would approach quantum theory as an approximation and a limiting case, "rather as the quantum theory itself approaches classical theory in the limit of high quantum numbers." (13)

Moreover, according to the orthodox interpretation of quantum theory the essential difference between quantum statistics and classical statistics is that in a classical situation statistical statements can be in principle avoided, whereas in quantum theory we are faced with the difficulty that the statistical elements cannot be avoided, not even in principle, because of the very special nature of quantum mechanical observations due to the unique relation between measuring apparatus and the object of measurement.

Classical probability statements do not involve interference terms of probability amplitudes, whereas in quantum theory we are faced with such interference terms because the Superposition Principle applies, and it is the Superposition Principle which brings out the non-local features of quantum theory.

Digression: In classical statistics one may express the information which one has of a system at a certain time through a probability distribution in phase space. The subdivision of phase space for classical statistics can be made arbitrarily small such that $\rho = (x_1, \dots; p_1, \dots)$ is the probability density that the system is in one phase point $(x_1, \dots; p_1, \dots)$. However, for quantum theory, the subdivision of phase space would, on account of the Indeterminacy Principle have to satisfy the condition $\delta x \delta p \geq \hbar$

The probability density ρ can be normalized by setting the integral over the whole of phase space equal to one. Thus

$$\int \rho \, d\Omega = 1 \qquad ; \; (d\Omega = dx_1, \dots ; \; dp_1, \dots)$$

If the probability distribution is given one computes the expectation value $\langle L \rangle$ of a classical ovservable $L = L(x_1, \dots ; p_1, \dots)$ through

$$\langle L \rangle = \int L \rho \, d\Omega$$

End of Digression

If alternative (c) is unacceptable and if the Indeterminacy of the other "half" of the conjugate variables is relative but not absolute because we can in fact gain information of them through different experimental arrangements, and also because the degree and kind of Indeterminacy of all conjugate variables is expressed in a complete and clear "picture" through the ψ function, what are we left with, asks Schrödinger? (14)

4. The New Epistemological Standpoint

The orthodox interpretation of quantum theory helps itself and us out of this difficulty says Schrödinger, "durch eine Zuflucht zur Erkenntnistheorie" and one teaches us that one ought not to differentiate between the real state of the physical object (Naturobjekt) and the knowledge that I have of it, or better perhaps, the knowledge I could have of it if I really tried. What is real "--so one says--are actually only perception (Wahrnehmung), observation (Beobachtung) and measurement(Messung)." (15)

The consequence is that we drop realism and accept some phenomenalistic position which is based on the indubitable thesis that what is real is only observation and measurement. Our scientific thinking is therefore based on only one thing,namely the results of measurements that are executable in principle. Our thinking cannot be based on any other kind of physical reality or model. Thus, all physical quantities must be measurable in principle, since the values of certain physical quantities is all we have that legitimately can be called real.

Therefore, if I have acquired in a given moment the best possible knowledge of the state of a system which the physical laws allow me to have, then any further question as to the real state is pointless, in so far as I am convinced that no further observation will give me further knowledge of the state of the system, at least

not without reducing my present knowledge by changing the system through further observation. (c.f.p. 71)

Moreover, Schrödinger points out that all numbers which appear in our calculations must be considered as numbers that represent measured values (Masszahlen). However, he continues, since we do not start completely from scratch, and are more than reluctant to abandon our mathematical apparatus, we are forced to "dictate from our desk which measurements are in principle possible, that is, must be possible in order to give sufficient support to our mathematical shematism (Rechenschema)." (16) This permits us, he continues, to obtain a sharp value for every variable. Therefore every variable must be measurable to an arbitrary degree of accuracy. We cannot be satisfied with less because we have given up realism. In order that we do not completely fall into skepticism we must hold on to something that is physically real. But that something which is physically real cannot be independent from observation and measurement. Therefore observations and measurements must be possible, if anything physically real is possible.

However, says Schrödinger, we have nothing but our mathematical apparatus by which to determine the best possible knowledge we may have of an object. Our mathematical apparatus tells us where Nature draws the "Ignorabimus Grenze" (17). And if we cannot obtain in practice the knowledge that our mathematical apparatus allows us to obtain, "then our measurement-reality (Messwirklichkeit) would surely depend on the industriousness or laziness of the experimenter. . . We must tell him, how far he can go, if only he was skilled enough. Otherwise it would seriously have to be feared that there, where we forbid further questioning, still something worth knowing may be asked about." (18)

5. Consequences of the New Epistemological Standpoint

Schrödinger compares the ψ function to an expectation catalogue, an "instrument" that predicts the probability of measured values. The ψ function completely describes the system in the sense that anything which can in principle be known about the system at time t is contained in the ψ function, our prediction catalogue. (19)

The ψ function is similar in many respects to the classical model and its state. It is "uniquely determined through a finite number of suitably chosen measurements on the object, half as many as were necessary in classical theory." (20) Secondly, the ψ function changes with time if it is left to itself, in the same way as the state of the model of an object in classical theory changes--deterministically. (21)

But if one makes another measurement our prediction catalogue, the ψ function changes abruptly. This discontinuous change, (Schrödinger calls it a jump (Sprung)), cannot, however, be predicted before the measurement has taken place <u>but depends on the obtained measured value itself</u>. It is this discontinuous change of the ψ function, as a consequence of further measurements, which demands the rejection of realism. It is for this reason also that we cannot think of the ψ function as an ideal conceptual model of a physical object, not because physical objects or systems never change abruptly, "but because from a realistic point of view the observation itself is a process of Nature and may not in itself evoke a disruption of the regular process of Nature." (22)

The rejection of realism has several logical consequences according to Schrödinger.

First Consequence

A physical quantity <u>has</u> in general no definite value before the measurement of the same. To measure a value of a physical quantity is not to <u>discover</u> or <u>find out</u> the value which it <u>has</u>. (23)

We have rejected <u>realism</u>, and have abandoned E.P.R.'s criterion or physical reality. Thus we cannot speak meaningfully anymore of a physical quantity as <u>having</u> a definite value which however we do not know before the measurement, but discover through measurement.

But then there must be some criterion other than C_b which helps us to decide whether or not a measurement is correct or incorrect, whether or not the results of our measurement are physically meaningful or are mere coincidental happenings in the laboratory. Since we rejected E.P.R.'s criterion of physical reality (C_b) how can we judge whether or not our measuring results correspond to physical reality without some sort of criterion C_b* as to what constitutes physical reality in the first place? That is, what could $C_b^* \neq C_b$ be, such that $[C_b^* \wedge \neg$ <u>Assumption A</u>$]$ is true, and at the same time provide a means by which to judge whether or not our measuring results have physical meaning? Now Schrödinger says:

> It is quite clear, if [physical] reality does not determine the measured value, then at least the measured value must determine [physical] reality, it [the measured value] must be really present <u>after</u> the measurement in the sense, which alone is still recognized. (24)

Therefore, Schrödinger says, the needed criterion is this: "<u>By repeating the measurement, the same result must be obtained</u>." (25)

He distinguishes between two kinds of repetitions of measurements:

a. Repetition of the first kind

Since a measurement of an observable generally evokes a discontinuous uncontrolable alteration of the ψ function or state vector, the state vector will immediately after the measurement coincide with the eigenvector corresponding to the eigenvalue obtained in the measurement, regardless of what the state vector was just prior to the measurment. (We assume for simplicity, non-degeneracy)

Therefore, if immediately after such a measurement we repeat the measurement such that no significant time evolution of the obtained eigenvector corresponding to the eigenvalue obtained in the preceeding measurement has taken place, then the same eigenvalue must again be obtained (within certain error limits), provided the measurement apparatus is somehow restored to its initial, neutral state before the second measurement; or another measurement apparatus is used instead which satisfies that condition.

Schrödinger calls this a control-measurement (Kontrollmessung) for which, as can readily be seen from the above discussion, the differentiation between the measurement instrument and the measurment object is crucial.

b. Repetition of the second kind

The difference between measurement object and measurement device is of minor importance here. When we perform a repetition of the second kind, we are checking so to speak the statistical predictions of quantum theory. We must therefore repeat the measurement on the system many times, and each measurement is performed with the system always in the same initial state $\psi_t(\underset{\sim}{r})$ just prior to each measurement. The same goes for the measuring apparatus, it too must always be in its neutral initial state before each measurement. Another procedure would be to measure instead a large number N, of individual but identical systems (e.g. atoms) all in the same dynamical state at the instant of measurement. Clearly a simultaneous measurement of N identical

independent systems each of which is in the same dynamical state, is in essence identical to N repeated measurements of one system always being in the same initial dynamical state $\psi_c(x)$ before each measurement.

Second Consequence

Compared to the classical model and its state, the information of the ψ function, our prediction catalogue, appears to be rather incomplete. It encompasses at most only 50% of the total description of a physical system. The rejection of realism, however, makes it incumbent upon us to consider the ψ function as a complete description. It must, says Schrödinger, on the basis of our new accepted epistemological standpoint, be impossible to obtain new information of the state of an object, which is not already contained in the function, since, as was pointed out previously, (c.f.p. 67) we have nothing but our mathematical apparatus by which to determine the best possible knowledge of the state of an object. (26)

If I have acquired in a given moment the best possible knowledge of the state of an object which my prediction catalogue allows me, then any further questioning for more information is pointless, <u>since the ψ function contains the maximum sum of possible knowledge of the system's state</u>.

We cannot therefore add knowledge to our expectation catalogue without erasing some knowledge. But to erase knowledge means that previous correct statements are now false. However, Schrödinger adds, a correct statement can only become false if the object (Gegenstand) to which it refers changes. Thus he concludes:

T_1: If there exist different ψ functions, then the system is in different states.

And if we speak only of systems for which ψ functions exist then the converse of T_1 says:

T_2: For the same ψ function, the system is in the same state. (27)

Third Consequence

Suppose S_I and S_{II} are completely separated and have never intereacted before, the interaction potential being zero. Suppose further, that each possess a complete expectation catalogue $|\varphi^I\rangle$ and $|\phi^{II}\rangle$ respectively. Sonce each expectation catalogue is a maximum sum of knowledge of each of the respective states of systems S_I and S_{II}, we also have, says Schrödinger, maximum sum of both systems together (i.e. S_{I+II}), "if we think that not each system individually but both together constitute the object (Gegenstand) of our attention and questioning with respect to the future." (28) Schrödinger continues:

> But the converse is not true. Maximum knowledge of the total system does not include necessarily maximum knowledge of its parts, not even then when both parts are totally separated from each other and do not influence each other at the time. It could be that a part of that, which one knows, refers to relations or conditions between the two component systems in the following way: When a definite measurement of the first system has this result, then for a definite measurement on the second system this and this expectation-statistic (Ewartungsstatistik) is valid. But if the measurement on the first system has another result, then for the second system another expectation (Ewartung) is valid. If a third result is obtained on the first system. then again another expectation on the second system is valid, and so on, . . . In this way some measurement process, or what is the same, some variable of the second system may be connected with the not yet known value of some variable of the first system and of course vice versa. If this is the case, if such conditional sentences (Konditionalsätze) are present in our total catalogue (Gesamtkatalog), then the total catalogue cannot have maximum sum of knowledge with respect to the component systems; because the content of two individual catalogues each catalogue containing maximum sum of knowledge of the respective component system's state would in itself be sufficient for a total catalogue, and conditional sentences could not still be added. (29)

In short, best possible knowledge of the composite system does not necessarily entail best possible knowledge of the component systems. That is, the composite system S_{I+II} is in a definite state, whereas the component systems considered individually, are not.

One is, however, tempted to say a priori that S_I or S_{II} must be in some definite state, but precisely which definite state we do not know. Saying this would correspond precisely to Assumption A. But on the basis of the new adopted epistemological standpoint and its consequences we are not allowed to say " I do not know the state of S_I and S_{II}", since the ψ function represents maximum sum of best possible knowledge of the state of a system. "If one has a ψ function," says Schrödinger, "then it may serve to describe the state of the system." (30) But, he continues, "The insufficiency of the function as a model-surrogate (Modellersatz) rests exclusively on the fact that we do not always have a ψ function in cases where we would expect to have one. " (31) But then one may not postulate that in reality there exists a ψ function but we have no knowledge of it and therefore the system is in a definite though unknown state. The newly adopted epistemological standpoint forbids this Schrödinger says. "She [the ψ function] is the maximum sum of best possible knowledge, and knowledge which no one knows is non." [Sie ist nämlich eine Summe von Kenntnissen und Kenntnisse, die niemand kennt, sind keine".] (32)

Therefore when two initially completely separate systems S_I and S_{II}, of whose respective states we have maximum knowledge through their respective ψ functions or state vectors $|\varphi\rangle, |\phi\rangle$ enter into a temporary interaction and at some time separate again, they can no longer be described in the same way as before. That is, S_I and S_{II} cannot be considered to be each in a definite though perhaps unknown state after the interaction.

Schrödinger says in another paper:

> I would not call that one but rather the characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought. By the interaction the two representatives (or ψ functions) have become entangled. To disentangle them we must gather further information by experiment, although we knew as much as anybody could possibly know about all that happened. Of either system taken separately, all previous knowledge may be entirely lost, leaving us but one priviledge: to restrict the experiments to one only of the two systems. (33)

The above discussion applies also directly to a quantum mechanical measuring process where a system S_O and a measuring instrument S_M interact for some time. The measurement process of the observable $\mathcal{L}$ with eigenvalue Λi consists in the fact that in a definite time interval Δt there is an interaction between S_O and S_M and a subsequent reading on S_M to see which eigenvalue of the observable $\mathcal{L}$ is obtained.

We suppose that before the actual measurement process S_O and S_M are completely separate and the interaction potential $H_\omega = 0$ and S_O and S_M each possess a complete expectation catalogue $|\varphi\rangle$ and $|\phi\rangle$ respectively. Since each expectation catalogue is a maximum sum of the best possible knowledge of the states of the respective systems, then we also have maximum knowledge of the composite system after, for example, interaction has taken place. But as was pointed out previously, the converse is not true. Best possible knowledge of the total system does not necessarily include best possible knowledge of the component systems. "The lack of knowledge" says Schrödinger, "is by no means due to the interaction being insufficiently known--at least not in the way that it could possibly be known more completely--it is due to the interaction itself." (34)

Digression: (35) To see this we write the Hamiltonian operator H of the composite system S_{o+M} as $H = H_o + H_M + H\omega$, where $H\omega$ describes only the interaction of S_o and S_M, and H_o and H_M are the respective Hamiltonian operators of S_o and S_M separately.

The state vector $|\varphi\rangle$ of the operator $\rho_{|\varphi\rangle} = |\varphi\rangle\langle\varphi|$ can be expanded in terms of the eigenvalue spectrum $|u_{\wedge i}\rangle$ of the operator $\mathcal{L}$ such that

$$|\varphi\rangle = \sum_i \langle u_{\wedge i}|\varphi\rangle |u_{\wedge i}\rangle \qquad (86)$$

and

$$1\,\rho_{|\varphi\rangle}1 = \rho_{|\varphi\rangle} = \sum_{j,K} |u_{\wedge i}\rangle\langle u_{\wedge i}|\varphi\rangle\langle\varphi|u_{\wedge k}\rangle\langle u_{\wedge k}| \qquad (87)$$

where $|\langle u_{\wedge i}|\varphi\rangle|^2 = \pi(\wedge i)$ is the probability that after the measurement the eigenvalue $\wedge i$ corresponding to $\mathcal{L}$ will be obtained.

Next we consider the composite system S_{o+M} which consists of the composite state vector

$$|\psi\rangle = |\varphi\rangle \otimes |\phi\rangle \qquad (88)$$

and

$$|\psi_i\rangle = |u_{\wedge i}\rangle \otimes |\phi\rangle \qquad (89)$$

For the state or projection operator of the composite system S_{o+M} we get from the above expansions, i.e. equations (88) and (89):

$$|\psi\rangle = \sum_i \langle u_{\Lambda i}|\varphi\rangle |u_{\Lambda i}\rangle \otimes |\phi\rangle \qquad (90)$$

$$= \sum_i \langle u_{\Lambda i}|\varphi\rangle |\psi_i\rangle \qquad (91)$$

and

$$\rho_{|\psi\rangle} = |\psi\rangle\langle\psi| = \sum_{i,k} |\psi_i\rangle\langle\psi_k| c_{ik} \ , \qquad (92)$$

where

$$c_{ik} \equiv \langle u_{\Lambda i}|\varphi\rangle\langle u_{\Lambda k}|\varphi\rangle^* \qquad (93)$$

Suppose at t = 0 the interaction between S_M and S_o begins and suppose it ends at time t = t', then we have $H\omega > 0$ for $0 \leq t \leq t'$. During this time interval the state vector of the composite system S_{o+M} changes. Therefore if

$$|\psi(t=0)\rangle = |\psi\rangle , \qquad (94)$$

then we get for $0 \leq t \leq t'$

$$|\psi(t)\rangle = e^{iH\omega t}|\psi\rangle \qquad (95)$$

and also

$$|\psi_i(t)\rangle = e^{iH\omega t}|\psi_i\rangle . \qquad (96)$$

After the interaction between the systems S_M and S_o has ceased, S_{o+M} is then in the state

$$|\psi'\rangle = |\Psi(t')\rangle \tag{97}$$

From equation (91) we write equation (97) as

$$|\psi'\rangle = \sum_i \langle u_{\lambda_i}|\varphi\rangle |\psi_i'\rangle, \tag{98}$$

where

$$|\psi_i'\rangle = |\psi_i(t')\rangle \tag{99}$$

and the state $|\psi_i'\rangle = \sum_i c_i |\psi_i(t')\rangle$ is again constant timewise for $t > t'$, and

$$\mathcal{P}_{|\psi'\rangle} = |\psi'\rangle\langle\psi'| = \sum_{i,k} |\psi_i'\rangle\langle\psi_k'| c_{ik} \; .$$

However, the state $|\psi(t')\rangle = \sum_i c_i |\psi(t')\rangle$ cannot anymore be written as a product of two states as in equation (88) where one state describes the object while the other state describes the measuring instrument. The state function of S_o and S_M have become, to use Schrödinger's terminology, entangled.

Now the aim of a measurement process is to be able to make a statement, about S_o alone. Since the system S_o interacted with S_M during Δt, S_o is connected with S_M in a complicated way. To make a statement about S_o alone it is necessary to isolate S_o from S_M. Formally this happens through a projection on to H^o, the Hilbert space which corresponds to the measurement object alone. (c.f.p. 25) However at this stage, the theory of the measurement process says nothing about the exact value of the physical quantity $\mathcal{L}$ but only indicates the

probabilities of the latter. The last step on the basis of which an exact statement about the state of the object can be made consists in taking a reading from the measuring instrument.

End of Digression

CHAPTER V

METHODOLOGICAL, METAPHYSICAL AND EPISTEMOLOGICAL JUSTIFICATIONS FOR CONSTRUCTING HIDDEN VARIABLE THEORIES

1. Intorductory Remarks

We saw in Chapter III that if Assumption A is untrue and must be rejected if quantum theory is correct, then E.P.R.'s criterion of physical reality C_b must also be rejected. But we also pointed out that it is only on the basis of experimental results and therefore on the basis of the correctness of quantum theory and on that basis alone that Assumption A and consequently E.P.R.'s criterion of physical reality is to be abandoned.

However, since the correctness of a theory does not necessarily entail its completeness, the question as to whether or not quantum theory is complete (that is, whether or not, as is usually assumed, the wave function does contain a complete description of "physical reality" of the system in the state to which it corresponds) is still left open.

We also saw, particularly in Chapter IV, that the orthodox interpretation is forced to adopt a new epistemological standpoint. It rejects Realism and adopts some phenomenalistic position which is based on the indubitable thesis that what is real is only observation and measurement.

Consequently, a physical quantity has in general no definite value before the measurement of the same, and the rejection of Realism and the adoption of this new phenomenalistic position compells us to consider the ψ function as a complete description. Thus, within the conceptual framework of this orthodox interpretation,

quantum theory is assumed to be complete or as complete as it can ever be. The orthodox interpretation assumes that the physical state of an individual system is completely specified and described by a wave function which, however, predicts only the probabilities of actual measured values that can be obtained in a statistical ensemble of similar experiments.

It is precisely this assumption which E.P.R. find objectionable. Moreover, this assumption cannot be tested experimentally, since it rests on the Phenomenalist's dogma that what is real is only observation and measurement. It therefore represents a trans-empirical or meta-empirical assumption whose truth or falsehood can only be investigated by going outside the conceptual framework of the orthodox interpretation of quantum theory. This Bohm and others attempt to do by seeking another interpretation of quantum theory in terms of hidden variable theories, which would allow us, at least in principle, to determine the precise behaviour of an individual system.

We shall attempt in this concluding chapter to give a qualitative account of Bohm's interpretation of quantum theory in terms of hidden variables. Furthermore, we shall support his methodological and epistemological reasons for attempting such an interpretation on the basis of a metaphysical analysis of dispositional properties of matter. We shall argue that we must be Realists and not Phenomenalists as far as dispositional properties are concerned.

Our support is, however, not only directed toward a specific hidden variable model as that given by Bohm, but is directed toward any attempt based on the idea of hidden variables in which a theory more general than present quantum theory is sought.

2. The Indeterminacy Principle

Bohm's new interpretation allows us to conceive "of each individual system as being in a precisely definable state, whose changes with time are determined by definite laws, analogous (but not identical with) the classical equations of motion."(1) Bohm continues:

> Quantum-mechanical probabilities are regarded (like their counterparts in classical statistical mechanics) as only a practical necessity and not as a manifestation of an inherent lack of complete determination in the properties of matter at the quantum level. (2)

However, those who support the usual interpretation of quantum mechanics do not assume that the Indeterminacy Principle may perhaps be a consequence of an inherent limitation of quantum theory in its present stage of development. They assume instead that the Indeterminacy Principle "represents an absolute and final limitation on our ability to define the state of things by means of measurements of any kind that are now possible or that will ever be possible." (3)

The Indeterminacy Principle can be derived in two ways. We may first of all start with the assumption already indicated, and which E.P.R. criticize, viz., that the wave function which determines only probabilities of concrete experimental results, nonetheless provides the most complete description of the quantum mechanical state of an individual system.

This assumption, together with de Broglie's relation $p = \hbar \underset{\sim}{k}$, where $\underset{\sim}{k}$ is the propagation vector, the value of which is the wave number associated with a particular Fourier component of the wave function, constitutes a sufficient basis on which the Indeterminacy Principle may be deduced. This procedure of arriving at the Indeterminacy Principle leads us to interpret it "as an

inherent and irreducible limitation on the precision with which it is correct for us even to conceive of momentum and position as simultaneously defined quantities." (4) Bohm further explains:

> For if, as is done in the usual interpretation of the quantum theory, the wave intensity is assumed to determine only the probability of a given position and if the $\underset{\sim}{k}$ th. Fourier component of the wave function is assumed to determine only the probability of a corresponding momentum $p = \hbar \underset{\sim}{k}$, then it becomes a contradiction in terms to ask for a state in which momentum and position are simultaneously and precisely defined. (5)

An alternative derivation of the Indeterminacy Principle is based on an analysis of the measurement process which consists of an interaction between the measurement object and measurement apparatus in terms of indivisible quanta, such that it is impossible to perform an actual measurement without creating some disturbance, no matter how sensitive the measuring device is. We may however ask whether this disturbance can be predicted or controlled, such that one could still in principle obtain unlimitedly accurate results, and therefore simultaneous measurements of both position and momentum. If this would be possible, at least in principle, then the Indeterminacy Principle would be violated and would not constitute an inherent and irreducable limitation within the conceptual structure of the theory, but a practical limitation that could be overcome, at least in principle, through improved measurement techniques.

However, since the Indeterminacy Principle is a necessary consequence of the completeness assumption of the orthodox interpretation of quantum theory (i.e. the wave function and its probability interpretation provide the most complete possible specification of the state of an individual system), Bohm points out (6) that Bohr and others have, in order to avoid a contradiction of the completeness assumption, made an additional assumption. This additional assumption asserts "that the process of transfer of a single quantum from

observed system to measuring apparatus is inherently unpredictable, uncontrollable, and not subject to a detailed analysis or description." (7) Bohm continues:

> With the aid of this assumption one can show that the same uncertainty principle that is deduced from the wave function and its probability interpretation is also obtained as an inherent and unavoidable limitation on the precision of all possible measurements. Thus, one is able to obtain a set of assumptions, which permit a self-consistent formulation of the usual interpretation of the quantum theory. (8)

Of course such a set of assumptions compell us, as we saw in the previous chapter, to renounce Realism and adopt a new epistemological standpoint which has its own logical consequences. Bohr has given a detailed and consistent analysis of this standpoint in terms of the Principle of Complementarity. This principle requires that we renounce the classical concept of models of physical systems, as well as mathematical models, that is, mathematical concepts that have a one-to-one correspondence with physical objects which they describe.

Consequently, as was pointed out in the previous chapter, the ψ function must not be conceived of as a conceptual model. The insufficiency of the ψ function as a model-surrogate (Modell-ersatz), we quoted Schrödinger as saying, (c.f.p. 73), rests exclusively on the fact that we do not always have a ψ function in cases where we would expect to have one. This simply means that the ψ function is not in a one-to-one correspondence with the behaviour of a system.

The Principle of Complementarity requires that we reject precisely defined conceptual models (pictorial or mathematical), and restrict ourselves to inherently imprecisely defined concepts, such as position and momentum, particle and wave, etc. The maximum degree to which we can define one member of a complementary pair of imprecisely defined concepts is reciprocally related to the definiteness of the other member. It is the experimental conditions which determine

how precisely every member of such a complementary pair of concepts can be defined. The measuring instrument needed for a precise measurement of momentum, for example, would preclude a precise and simultaneous measurement of position.

Thus, together with the completeness assumption, the Principle of Complementarity "implies a corresponding unavoidable lack of precision in the very conceptual structure, with the aid of which we can think about and describe the behaviour of the system."(9) This means that "the basic properties of matter can <u>never</u> be understood rationally in terms of unique and unambiguous models . . . the limitation on our concepts implicit in the principle of complementarity are regarded as absolute and final." (10)

The consequences are, as Bohm further points out, that we are not allowed, for example:

> . . . to attempt to describe in detail how future phenomena arise out of past phenomena. Instead, we should simply accept without further analysis the fact that future phenomena do in fact somehow manage to be produced, in a way that is, however, necessarily beyond the possibility of a detailed description. The only aim of a mathematical theory is then to predict the statistical relations, if any, connecting these phenomena. (11)

3. Criticism of the Orthodox Interpretation of Quantum Mechanics

Bohm's criticism of the usual interpretation of quantum theory is particularly directed against the latter's insistence to "give up the possibility of even conceiving precisely what might determine the behaviour of an individual system at the quantum level, without providing adequate proof that such a renunciation is necessary." (12)

The orthodox interpretation of quantum theory is self-consistent, but such consistency does not constitute in itself sufficient grounds "to exclude the possibility of other equally consistent interpretations, which would involve additional elements or parameters permitting a detailed causal and continuous description of all processes, and not requiring us to forego the possibility of conceiving the quantum level in precise terms." (13)

This insistence of the orthodox doctrine is furthermore supported by von Neumann's theorem according to which the assumption of "hidden variables" is incompatible with established results of quantum theory. According to von Neumann's theorem, Bohm points out:

> . . . it would not only be impossible to verify experimentally any causal theory that aimed to predict the detailed behaviour of an individual system at the atomic level, but it is impossible even to conceive of such an explanation. For von Neumann proved that no conceivable distribution of motion of 'hidden' parameters in the observed system could lead to precisely the same results as those of Schrödinger's equation, plus the probability interpretation of the wave function. Thus, for example, one could no longer imagine that even a Laplacian super-being who obtained information without disturbing the system by means of a measurement could make precise predictions about the future. In this way the indeterminacy principle is supplemented; for the impossibility of making measurements more precise than the limits set by this principle should, according to the theorem of von Neumann, be regarded as a result of the fact that nothing even exists corresponding to a set of 'hidden' parameters having a degree of precision of definition going beyond these limits. Thus the renunciation of causality in the usual interpretation of the quantum theory is not to be regarded as merely the result of our inability to measure the precise values of variables that would enter into the expression of causal laws at the atomic level, but, rather, it should be regarded as a reflection of the fact that no such laws exist. (14)

Von Neumann's proof consists partly in a showing that on the basis of the usual rules in quantum theory for calculating the probabilities there can be no "dispersion-free states", that is, to quote Bohm:

> . . . states in which the values of all possible observables are simultaneously determined by the physical parameters associated with the observed system. For example, if we consider two non-commuting observables, p and q, then von Neumann shows that it would be inconsistent with the usual rules of calculating quantum-mechanical probabilities to assume that there were in the observed system a set of hidden parameters which simultaneously determined the results of measurements of position and momentum 'observables'. (15)

Bohm agrees with this, but suggests in his new interpretation, that the "observables" are not properties which belong to the observed system alone, but also to the measuring apparatus such that "when we measure the momentum 'observable', the final result is determined by hidden parameters in the momentum-measuring device as well as by hidden parameters in the observed electron." (16)

Bohm concludes:

> Von Neumann's proof . . . that no single distribution of hidden parameters could be consistent with the results of quantum theory is therefore irrelevant here, since in our interpretation of measurement of the type that can now be carried out, the distributionof hidden parameters varies in accordance with the different mutually exclusive experimental arrangements of matter that must be used in making different kinds of measurements. In this point, we are in agreement with Bohr, who repeatedly stresses the fundamental role of the measuring apparatus as an inseparable part of the observed system. We differ from Bohr, however, in that we have proposed a method by which the role of the apparatus can be analyzed and described in principle in a precise way . . . (17)

However, the question whether or not there exists a deeper, or what Bohm calls a sub-quantum-mechanical level of continuous and causally determined motion which would lead to the present laws of quantum mechanics as an approximation, is still problematic. For example, Capasso, Fortunato and Selleri point out in a recent paper that to arrive at his conclusion von Neumann "has tacitly assumed that his hypotheses are so general that they will be satisfied by any theory having Q.M. has an approximation" (18) Moreover, Bohm and Bub point out that it has been suggested that:

> . . . the reasoning involved in von Neumann's proof is circular and that the conclusion is tacitly assumed in the premises on which the argument is based. Other authors (the most recent of whom are Jauch and Piron 19) deny that the conclusion depends on a circular argument . . . Finally, it has recently been pointed out by Bell 20 that von Neumann's proof is based on certain unnecessarily restrictive assumptions and that when these are not made the proof breaks down. (21)

Nonetheless, to simply assert that there is no deeper level of causally determined motion is, as Bohm rightly points out, "just a piece of circular reasoning, since it will follow only if we assume beforehand that no such level exists." (22)

It seems that on the basis of the Indeterminacy Principle and von Neumann's proof conclusions have been drawn which follow neither from empirical facts nor from the mathematical concepts of the theory, but which are based on the usually implicit assumption that the current formulation of quantum theory is final and absolute in the sense that it will not be superceeded by deeper theories-- in which case the present theory would constitute a good appriximation on the atomic scale. The Indeterminacy Principle would, then, have no relevance at the sub-quantum-mechanical level since the assumptions on which it rests would break down at that level.

We therefore suggest that from methodological and epistemological considerations alone, it is incumbent upon us to inquire whether or not the statistical fluctuations have causes which are however as yet unknown to us, but which might be explained through "deeper" theories. To postulate hidden variables would be analogous to early forms of atomic theories in which the existence of atoms was postulated in order to explain large-scale phenomena. Bohm says that in the case of Brownian motion, for example,

> . . . the postulate was made that the visible irregular motions of spore particles originated in a deeper but as yet invisible level of atomic motion. Hence all the factors determining the irregular changes in Brownian motion itself, but rather, most of them were assumed to exist at the level of atomic motions. (23)

The assumptions that underlie the orthodox interpretation of quantum theory are according to Bohm supported by two basic considerations.

One believes first of all that man is capable "to conceive of only two kinds of things, namely fields (24) and particles." (25) Since we can only conceive of what we encounter in everyday experience, " or at most in experience with things that are in the domain of classical physics where . . . all phenomena fall into one or the other of these two classes", (26) it is believed that in the classical domain where neither particle concept nor the wave concept each by themselves provide an adequate description of matter, we have "passed beyond the domain of what we can conceive of." (27) Heisenberg, for example, expresses this point of view as follows:

> Light and matter are both single entities, and the apparent duality arises in the limitation of our language. It is not surprising that our language should be incapable of describing the processes occurring with the atoms, for, as has been remarked, it was invented to describe the experiences of daily life, . . . Furthermore, it is very difficult to modify our language so that it will be able to describe these atomic processes, for words

> can only describe things of which we can form mental pictures and this ability, too, is a result of daily experience. (28)

C. F. von Weiszäcker argues on similar lines when he says:

> How can we experience something of the atoms? In the last resort only through sensuous perception, therefore in the language of classical physics . . . Classical Physics provides the character and mode in which the atom alone can appear; it is because of this that it says nothing about the atom in itself. (29)

According to this view we must be satisfied with statistical results obtained through suitable mathematical calculations, since our conceptual thinking is restricted to the classical domain only. "As a result," Bohm remarks, "any effort at conceiving of a sub-quantum-mechanical level is foredoomed to failure, since even if such a level should actually exist, we could never have direct experience with the entities in it, and could, therefore, never hope to imagine what these entities might be like." (30)

However, objections of this kind to hidden variable theories do not stand on firm grounds. As was pointed out in the previous chapter, scientific progress is essentially reconstruction. It is not a mere passive reflection upon experiences we have and therefore a passive accumulation of data.

It is true that we start out with everyday experience, and that our knowledge and thinking is guided by it, but this does not necessarily imply that knowledge is derived from sense experience itself, for as Kant says:

> Experience is, beyond all doubt, the first product to which our understanding gives rise, in working up the raw material of sensible impressions. (sinnliche Empfindungen) Experience is therefore our first instruction, . . . Nonetheless, it is by no means the sole field to which our understanding is confined. Experience tells

> us indeed, what is, but not that it must necessarily be so, and not otherwise. It therefore gives us no true universality; and reason, which is so insistent upon this kind of knowledge, is therefore more stimulated by it than satisfied. (31)

It is impossible that only passive reflection of everyday experiences, or even only the accumulation of data within a laboratory, could have brought about scientific progress. Aristotle's theory of motion, for example, had a great deal of support from everyday experience. In order to make progress scientifically we must go beyond physical phenomena and attempt to learn more than what the phenomana themselves suggest.

The other major consideration which supports the assumptions of the usual interpretation of quantum mechanics and opposes hidden variable theories, is based on the positivistic bias which asserts that one ought not to postulate the existence of entities which cannot already be observed by acceptable methods. This objection is based primarily on the Verification Principle (32) which states in part, that only those statements are meaningful which can be empirically verified, and that therefore only meaningful statements may be either true or false. Statements that are not subject to verification and falsification that determine their truth or falsehood are considered to be meaningless statements.

We may counter this objection in three ways: First, the Verification Principle is itself empirically unverifiable and by its own criteria doomed to be meaningless. Secondly, the Positivist's principle is not a good working hypothesis within the context of scientific research. One can give many examples from the historical development of science in which it was useful to postulate the existence of unknown, unobservable entities, long before any acceptable procedures were available that permitted their direct observation. It is often from speculative methods that useful results will be obtained. It

is in this way that de Broglie, for example, obtained the wave equation which was supported by experiments only afterwards.

Thirdly, the emphasis should not be put on the verifiability of a theory in the first place, but rather, as Popper has suggested quite cogently, the basic requirement for a scientific theory is that it should be constructed and expressed in terms that make it falsifiable. (33)

It appears that the acceptance of the completeness assumption, the Complementarity Principle and von Neumann's proof, together with the empirical success of quantum theory, makes it not only impossible to construct deeper theories which embody a more general structure of concepts, but prevents one even from asserting the possibility that the basic assumptions and postulates underlying quantum theory may be false. As Bohm remarks: "The usual interpretation therefore presents us with a considerable danger of falling into a trap, consisting of a self-closing chain of circular hypotheses which are in principle unverifiable if true." (34)

Can we accept in earnest the claim that the postulates of quantum theory are final and absolute truths that can never be falsified or shown to be valid only as approximations or limiting cases in the atomic domain--and that therefore the only changes of the theory to be made are in terms of ad hoc extensions?

In view of this question Bohm draws the following pertinent analogy:

> What is common to both classical physicists and modern physicists is, therefore, a tendency to assume the absolute and final character of the general features of the most fundamental theory that happens to be available at the time at which they are working. Thus, the usual interpretation of quantum theory represents, in a certain sense, a rather natural continuation of the mechanistic attitude of classical physicists, suitable adjusted to take into account the fact that the most fundamental theory now available is probabilistic in form, and not deterministic. (35)

Hidden variable theories may serve two principle purposes according to Bohm. (36) Firstly they provide a specific form within which to crystallize the limitations and presuppositions of the orthodox interpretation, and are a good testing ground for the latter's assumptions and postulates. Secondly, hidden variable theories may shed light on problems that are not very well understood. For example Bohm points out:

> At distances of the order 10^{-13}cm. or smaller and for times of the order of this distance divided by the velocities of light or smaller, present theories become so inadequate that it is generally believed that they are probably not applicable, except perhaps in a very crude sense. (37)

However, some physicists would argue that in order to overcome some of the problems we may be led to new theories, that would in all likelihood turn out to be even more probabilistic than present quantum theory is. Although this possibility cannot be denied a priori, it represents a rather narrow and pessimistic attitude which is again based on the assumption that the present theory cannot be superceeded by a more fundamental theory which would povide an objective and precisely definable description of physical reality at the quantum level of accuracy. Moreover, in line with a previous quotation (c.f.footnote 35) Bohm points out:

> . . . nineteenth-century physicists could equally well have claimed that the unbroken success of the deterministic laws of classical mechanics in three centuries of application was a very strong indication that progress into new domains would be very likely to lead to laws that were, if anything, even more deterministic than those that already existed. (38)

The historical development of science therefore indicates that our epistemology is to a large extent influenced by present theories, and Bohm asserts quite rightly, that it is therefore "not wise to specify the possible forms of future theories in terms of purely epistemological limitations deduced from existing theories." (39)

4. An Analysis of Dispositional Properties

The subjectivistic, positivistic or phenomenalistic interpretations of quantum theory assert that a micro-object has no properties at all when it is not observed. According to this view, "one may say that its only mode of being is to be observed; for the notion of an atom existing with uniquely definable properties of its own even when it is not interacting with a piece of observing apparatus, is meaningless within the framework of this point of view." (40)

Digression: Phenomenalism as used in the present context must be distinguished from the orthodox metaphysical Phenomenalism which reduces everything to sense data. In the present context a Phenomenalist can be a Realist about large-scale phenomena but denies the real (autonomous) existence of well defined properties of a microscopic object.
End of Digression

A Realist's account of dispositional properties of an object "X" can be illustrated in the following way: To speak of a piece of sugar as having the dispositional property of solubility entails that the piece of sugar is in some non-dispositional state, or has some categorical basis (e.g. molecular structure) which is responsible for the sugar manifesting the behaviour of dissolving in a certain circumstance such as the sugar's immersion in water, a manifestation of behaviour whose nature (namely, that of dissolving)

makes the dispositional property the specific dispositional property it is. (41)

The last part of the sentence may appear to be redundant, but it is not. An object "X" which possesses a certain non-dispositional property "M" may display, whenever subjected to a condition "C" behaviour B_1, B_2, . . . B_n respectively. Thus, for example, a piece of iron may be disposed to expand, to change colour, and to become softer whenever it is heated.

However, according to the Phenomenalist, the sugar's possession of a dispositional property does not entail that the sugar is in a particular state, or undergoes a particular change, but that the sugar is bound or liable to be in a particular state, or to undergo a particular change, namely, dissolving, when a particular condition is realized, namely the immersion of the sugar in water.

Thus for the Phenomenalist, the object's possession of a dispositional property <u>does not entail</u> the existence of a categorical state. Perhaps this distinction may best be illustrated as follows:

If P_1, P_2, . . . P_n stand for the dispositions of an object "X", and the object "X" exhibits under a certain condition "C" the characteristic behaviours B_1, B_2, . . . B_n, we may, in principle at least, determine in two ways whether a given object "X" has the dispositional properties P_1, P_2, . . . P_n at a given time "t". The <u>Phenomenalist's method</u> consists in producing a specified condition "C" and in determining whether or not the behaviours B_1, B_2, . . . B_n are obtained. The <u>Realist's method</u> consists in analyzing the "<u>state</u>" of "X" at time "t" in sufficient detail, (<u>this may however not always be possible in practice, but it is assumed to be possible in principle</u>), such that it is possible to deduce from the information

gained of the "<u>state</u>" of "X" (viz. the categorical basis of "X") with the help of scientific laws, the behaviours B_1, B_2, . . . B_n which "X" would exhibit under any specified circumstance "C". The Realist would then be able to predict whether or not under a specified condition "C", "X" would exhibit behaviours B_1, B_2, . . . B_n and consequently, whether or not "X" possesses P_1, P_2, . . .,P_n.

For example, let "X" be a lump of sugar and "P" be its dispositional property of solubility which is manifested through its behaviour when immersed in water. In principle, at least, the hypothesis that the piece of sugar possesses the dispositional property "Solubility" is tested in two ways. The Phenomenalist's method consists in immersing the sugar into water and observing its behaviour under this particular circumstance "C". If it dissolves, it has the dispositional property "Solubility". The Realist, on the other hand, may study the molecular structure of the piece of sugar, and with the help of physical laws, may predict the behaviour which would result under the specified condition.

Applying the Realist's account to such phenomena as power, forces, capacity, etc., we may, for example, say that a magnet has the disposition or capacity to attract certain metals, and that this capacity to attract is a causal factor in the process of attraction, since this capacity or disposition to attract is contingently related to microscopic events within a magnet whose electrons have synchronized spins. But the magnetic intensity may vary with time. In fact, what was at time t_1 a magnet may not be a magnet at time t_2. The Phenomenalist is thus unable to predict whether or not a piece of iron has the disposition of magnetism at t_2, even though it manifested this property at t_1. However, the Realist may overcome this limitation and take advantage of the properties of matter which in this case depend significantly on the specific microscopic structure of the particular object in question.

He may, for example, examine the microscopic structure through a photomicrograph. A photomicrograph, made by reflecting polarized light from the surface of a piece of metal, can visually indicate the metal's magnetic properties. (42) Analogously, we may have to, and may even be able to take advantage of the properties of matter which depend significantly on the sub-quantum-mechanical level, in order to overcome the limitations of present quantum theory.

Moreover, the latter example of the magnet illustrates that the Postivist's assertion, that we should not postulate the existence of entities which we do not already know how to observe, imposes severe restrictions on scientific progress. Without assuming that certain entities might be real we would have no real cause for searching for particular methods specifically designed to show that they are real. It is simply not the case that scientific progress is made only through discovery of new brute facts. New facts lead to new concepts and theories which in turn lead to new experiments and thus to the discovery of new facts.

But the Phenomenalist and the Realist agree that the meaning of dispositional predicates is such that it is logically possible for an object to have a disposition which is nonetheless never manifested in actual behaviour.

Furthermore, both would agree what it is to be a dispositional property, but they would disagree on what <u>having</u> a dispositional property entails. Both agree that to say <u>"X" has a dispositional property</u> entails <u>there is behaviour "B" and circumstance "C"</u> such that <u>"X" would exhibit "B" in "C"</u>.

Analogously, both the Realist and the Phenomenalist would agree that "X" (= electron) has the dispositional property to be particle-like in that "X" exhibits particle-like behaviour B_p in a certain circumstance C_p, and that it has the dispositional property

P_W to be wave-like in that it exhibits wave-like behaviour B_W in a different experimental arrangement C_W. Note, that C_p and C_W are mutually exclusive experimental arrangements.

But the Realist wants to say more than that. He wants to talk about dispositions in terms of inner states of objects and their causal relation to specific manifest behaviour. The hidden-variable theorist likewise wants to talk, for example, about the dispositions of an electron, which under mutually exclusive experimental arrangements exhibits either wave-like or particle-like behaviour, in terms of underlying phenomena.

The Realist, when talking about dispositions, deals not only with possibilities but also with actualities, whereas the Phenomenalist deals with possibilities only. For the Realist to talk about dispositions entails talking about the existence of some non-dispositional state. Thus, dispositions can be identified contingently with states and events. The Phenomenalist, on the other hand, thinks of dispositions only in terms of possible behaviour.

According to the Realist, having the dispositional property "P" is not sufficient for "X"'s doing "B" in "C". Likewise, for the hidden-variable theorist, having the dispositions to be wave-like (P_W) and particle-like (P_p) is not sufficient for "X"'s ("X" = electron) doing B_W in C_W or B_p in C_p. What is required and sought is a further property "Q" (categorical basis) which is responsible for "X" displaying "B" in "C".

This can be stated more generally and precisely as follows:

D: "P" is a dispositional property if and only if there is behaviour "B" in circumstance "C" and a non-dispositional property or state "Q" such that the following is true: An object "X" has "P" if and only if "X" has "Q" and "X"'s having "Q" makes "X" exhibit "B" in "C".

But this definition seems to be circular since we use the term "non-dispositional" to define "dispositional". But the former term itself is undefined so long as the latter term is not defined. The question arises whether or not this circularity can ever be completely avoided, that is, whether or not it is possible to make an <u>absolute</u> distinction between a non-dispositional property and a dispositional property, and therefore define or reduce the latter in terms of the former in an absolute sense.

The answer is no. We would want to contend that it is impossible to reduce in the final analysis a dispositional property to a non-dispositional property or predicate, for it can be shown that all physical as well as psychological properties are dispositional.

<u>Digression</u>: What is often overlooked is that not only predicates that have a suffix like "able", "ible", etc. are disposition predicates, but that every predicate that describes a quality or property of an object or a system is a dispositional predicate. Quine says that it is only "on this etymological count, if at all, that such terms as 'red' might not be said to be dispositional as well" (43) Goodman maintains that "to say that a thing is hard, quite as much as to say that it is flexible, is to make a statement about potentiality . . . and for that matter, a red object is likewise one capable of certain colour appearances under certain lights, and a cubical object is one capable of fitting try squares and measuring instruments in certain ways." (44)

Popper likewise suggests (45) that the dispositional character of any universal property of an object will become clear if we consider what tests we should undertake when we are in doubt as to whether or not the property is present in some particular case. Thus red is dispositional in that it is able to reflect a certain kind of light. Something looks red in certain situations or as Quine puts it, "an object is red if it is disposed, given a chance to reflect a certain range of low frequencies selectively" (46) Even "looking

red" is dispositional, for, according to Popper (47) it describes the disposition of a thing to make observers agree that it looks red. He rejects the distinction between dispositional and non-dispositional terms on the grounds that "breakable" and "broken", "soluble" and "dissolved" are all dispositional, but the difference is one of degree. "Breakable" is "dispositional" in a higher degree than "broken", but nonetheless both are dispositional.

Dispositional properties characterize the law-like behaviour of objects. In attributing the universal term "glass" to a pane, one specifies certain properties of the pane, viz, being brittle, translucent, etc.. Each of these properties is a manifestation of the disposition "glass" that characterizes the "law-like" behaviour of glass.

End of Digression

To make the above definition a bit clearer, all we can do is to define "Q" phenomenalistically in changing "Q" is a non-dispositional property to "Q" is a property other than the property of being such that an object would exhibit "B" in "C". Therefore

D*: "P" is a dispositional property if and only if there are "B" and "C" and a property "Q" other than the property of being such that an object would exhibit "B" in "C", such that the following is true: An object "X" has "P" if and only if "X" would exhibit "B" in "C" and there exists "Q" such that QX, and that QX makes "X" exhibit "B" in "C".

Therefore a dispositional term like soluble can never be conclusively defined operationally nor in any other way. If we grant that all universals are disposition terms, that is, if we transcend the etymological claim that suffixes determine the dispositionality of words, we will find that for example, the predicate "soluble" cannot be reduced to non-dispositional terms. Carnap's (48) "reduction sentence"-- $(x)\left[C(x) \rightarrow \left[S(x) \leftrightarrow B(x)\right]\right]$ -- (i.e. if "X" is put into water

then "X" is soluble in water if and only if it dissolves) reduces the newly introduced concept S (read solubility) to the observation predicates "B" and "C", which are themselves capable of dissolving "X"; and to say that "X" has dissolved is to say that it has not disappeared, which implies that "dissolved" itself is a dispositional predicate, indicating that "X" is "recoverable".

As Realists we should want to avoid the circularity of operational definitions couched in a phenomenalistic language. It prevents the danger of succumbing to total skepticism about dispositions, as is the case for the Phenomenalist, whose certainty about dispositional properties actualize themselves.

Phenomenalistic statements or propositions are the least uncertain because they are being the least commital in what they assert, and for this reason alone, they are not interesting and consequential enough from a scientific point of view.

Yet we must concede, that even as Realists, we cannot reduce dispositional terms or properties to non-dispositional ones in the absolute sense. The distinction we do draw between a categorical basis and the corresponding dispositional property or properties is relative and depends on the kind of dispositional property or properties we deal with. But nonetheless, making this distinction represents a good working hypothesis within a scientific context as was shown by previous examples.

The legitimacy of inferring from the categorical basis to the dispositional property and then to the behaviour of an object, the legitimacy of prediction in general, is based on the Realist's assertion that there is a contingent relation between the categorical basis and the dispositional property of an object. The Realist must therefore accept the Principle of Sufficient Reason in order to relate the categorical basis to the dispositions of an object in this fashion.

5. Wave-Particle Duality Phenomenon Interpreted in Terms of a Hidden Variable Model and Dispositional Properties

Usual Interpretation of the Double-Slit Experiment

Consider a source S of micro-objects X (e.g. X = electron) which is so weak that electrons are sent separately and independently along the x-axis which is perpendicular to the screen I containing the two slits A and B. (c.f. Fig. 1)

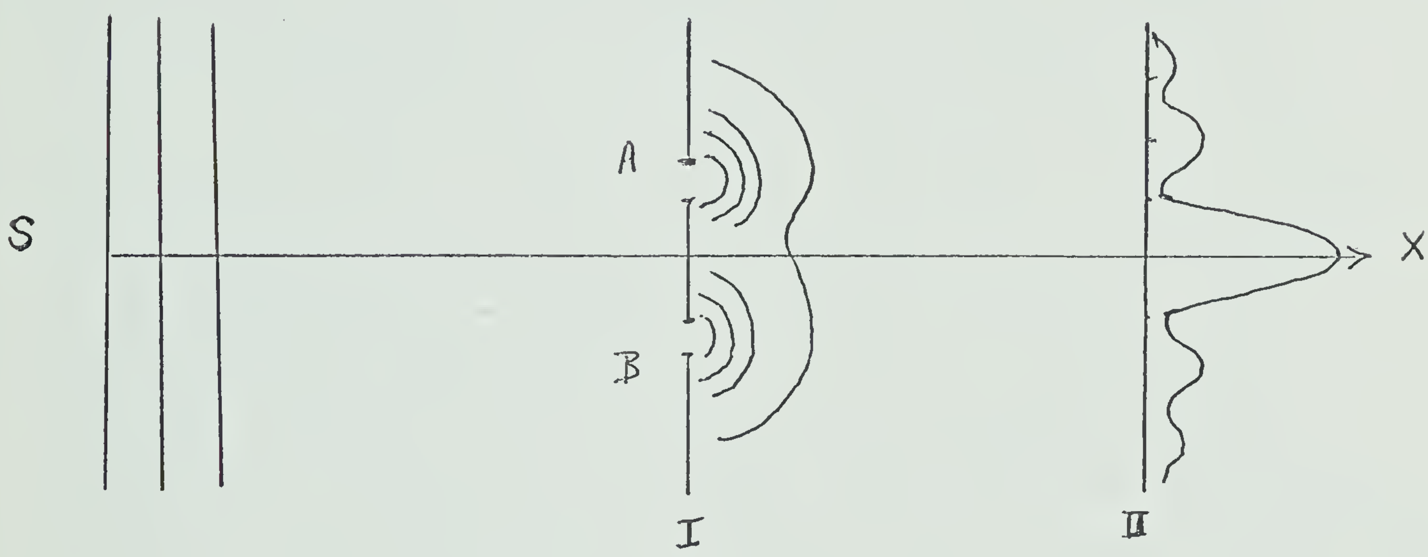

Fig. 1

We assume that all electrons have the same initial momentum and therefore the same wave function in the term of a plane wave, i.e. $\psi = exp\left[\frac{i}{\hbar} px\right]$ (40)

According to the usual interpretation of this experiment, the electron has no trajectory and position at the left side of screen I and is only represented by the plane wave ψ. On screen I the plane wave ψ will diffract into two cylindrical waves centred on the two slits A and B. At the right side of screen I the waves interfere and produce on Screen II an interference pattern of high and low intensities which determines the experimental distribution $P = |\psi|^2$

of the impacts of the particle-like aspect of the electron. That is, at screen II the electron's dispositional property to be particle-like is manifested.

However, at screen I the electron manifests its dispositional property to be wave-like since the interference pattern at screen II is influenced through the existence of the two slits A and B of screen I.

According to the usual interpretation of quantum theory we need two conceptions to characterize a micro-object,--the wave-conception and the particle-conception. Now neither wave-concept nor particle-concept is by itself capable of giving a full and satisfactory description of matter, nor is it possible according to the usual interpretation of quantum theory, that both concepts can simultaneously represent the same micro-object. (e.g. electron) Our knowledge of the dispositional properties of the same electron to be particle-like or wave-like is therefore reduced to utter skepticism except on occasions when one of the dispositional properties manifests itself because a correspondingly suitable experimental arrangement is realized. The orthodox interpretation is driven to this phenomenalistic position since it asserts that the same electron does not possess objectively both dispositional properties simultaneously.

As a consequence, quantum theory does not provide an objective and precisely defineable description of the behaviour of independently moving and precisely defined micro-objects, but it constitutes a formalism which predicts the probability of measured values of experiments. Bohm and Bub state: "At best then, the quantum theory can be regarded as an elaborate system of algorithms for computing the probabilities of experimental results." (50)

Moreover, the knowledge that can be obtained in quantum theory is statistical in character, but not in the classical sense, since the probabilities we are dealing with in quantum theory do not

result because of our ignorance of some more detailed behaviour (e.g. categorical basis), but represent "lawless" distributions. That is, probabilities in quantum theory must be differentiated from probabilities in classical statistical mechanics in that quantum-mechanical probabilities reduce to mere contingencies. For this reason statistical knowledge in quantum theory is ultimate knowledge irreducible (not even in principle) to a more precise knowledge that may take into account hidden phenomena, or hidden variables. At least that is what the orthodox interpretation of quantum theory wants us to believe.

New Interpretation Based on a Hidden Variable Model and Dispositional Properties

Instead of saying that the micro-object is either wave-like or particle-like, because it manifests the dispositional properties to be wave-like and to be particle-like under mutually exclusive experimental arrangements, de Broglie, Vigier, Bohm and others assume that it is objectively both a wave and a particle. That is, they assume that the same electron possesses simultaneously the dispositional properties to be wave-like and particle-like, just as in an analogous fashion a lump of sugar possesses simultaneously the dispositional properties of solubility, hardness, sweetness etc. As is evident from the foregoing discussion, to make such an assertion entails that one adopts a Realist account of dispositional properties as discussed above. (c.f.p.93-9)

The essential features of this new interpretation of quantum theory are according to Bohm (51) as follows:

(a) With each micro-object (e.g. electron) is associated a body confined in a small region of space. It is assumed to be smaller than the size of an atom and for all purposes it can be approximated at the atomic level by a mathematical point.

(b) It is assumed that associated with this body is a wave, without which the body can never be found.

Digression: A quote from a recent paper by Capasso, Fortunato, and Selleri may illustrate the above remarks more clearly:

> We start from the following realistic postulate: An elementary particle is always associated to a wave objectively existing. This postulate is admittedly rather vague. The only new fact is that the wave is postulated as objectively ["Objectively is taken to mean: independently for all observers] existing. This is certainly in contradiction with Q.M., where even the particle, let alone the wave cannot be assumed to be objectively existing. What we claim is that a theory developed starting from the realistic postulate leads to predictions different from those of Q.M. only for experiments which have never been done . . . Further comments about the postulate are the following: The association of the wave ψ and particle must be such that the probability density for observing the particle is given by $|\psi|^2$, the familiar result of Q.M. The wave has to be thought of as a real entity in some kind of postulated medium. Thus wave and particle are reminiscent of a boat in a lake. Boat and wave are both objectively existing and are found to be associated, in the sense that you cannot find a boat without a wave; the opposite is, however, possible. (52)

End of Digression

(c) It is assumed that the wave associated with the body will be an oscillation in a new kind of field represented mathematically by the ψ field of Schrödinger.

(d) The Schrödinger wave function represents an objectively real field similar to gravitational and electromagnetic fields, having some new characteristics of its own. It now represents not just a mere mathematical formalism that calculates certain probabilities, but is an objectively real field.

(e) The field satisfies Schrödinger's equation which consists of a partial differential equation that determines the future changes of the objectively existing field in terms of its value at all points in space at a given instant of time.

(f) It is assumed that the field and the body are connected in such a way that the ψ field exerts a new kind of "quantum-mechanical" force on the body.

(g) It is also assumed that the body may exert a reciprocal influence on the ψ field. But this reciprocal influence is so small that it may be neglected in the quantum-domain but is likely to be significant in the sub-quantum domain. [the categorical basis] .

(h) We need not be overly concerned with the precise nature of the quantum-mechanical force which the ψ field exerts on the body and the reciprocal influence on the ψ field by the body. All that is necessary is that the quantum mechanical force is such that it produces a tendency to pull the body into regions where $|\psi|$ is largest.

(i) This tendency is supplemented by random motions undergone by the body, "motions which are analogous to the Brownian movement". (53) This random motion could for example "come from the fluctuations of the ψ field itself. Indeed, it has been characteristic of all other fields known thus far that typical solutions to the field equations represent in general only some kind of average motion . . . Hence, it is not unreasonable to suppose that the ψ field is undergoing random fluctuation about an average that satisfies Schrödinger's equation and that these fluctuations communicate themselves to the body. The details of these fluctuations would then represent properties of the field associated with a sub-quantum mechanical level the categorical basis , since the quantum-mechanical level is treated in terms of the mean part, which satisfies Schrödinger's equation. On the other hand, the

bodies could obtain a random motion from a sub-quantum level categorical basis in other ways, for example, as in Brownian motion, by direct interaction with new kinds of entities existing in this lower level." (54)

It is not relevant to know at this stage where the fluctuations come from and what precisely they consist of. From a realistic standpoint we assume that there exists a categorical basis that is responsible for the manifestation of the respective dispositional properties, although we may have no precise knowledge as yet of its specific structure. Thus in the present context we assume that these fluctuations exist at the sub-quantum-mechanical level (categorical basis) although their precise nature is as yet not known.

(k) These postulated fluctuations "will produce a tendency for the body to wander in a more or less random way over the whole of space accessible to it. But this tendency is opposed by the 'quantum-force' which pulls the body into places where the field is most intense. The net result will be to produce a mean distribution in a statistical ensemble of bodies, which favours the regions where the field is most intense, but which still leaves some chance for a typical body to spend some time in the places where the ψ field is relatively weak." (55)

Bohm concludes:

> . . . instead of starting from Born's probability distribution [$P = |\psi|^2$] as an absolute and final and unexplainable property of matter, we have shown how this property could come out of random motions originating in a sub-quantum mechanical level [i.e. the categorical basis] (56)

On the basis of this new interpretation of quantum theory, the wave-particle duality phenomenon can now be explained as follows:

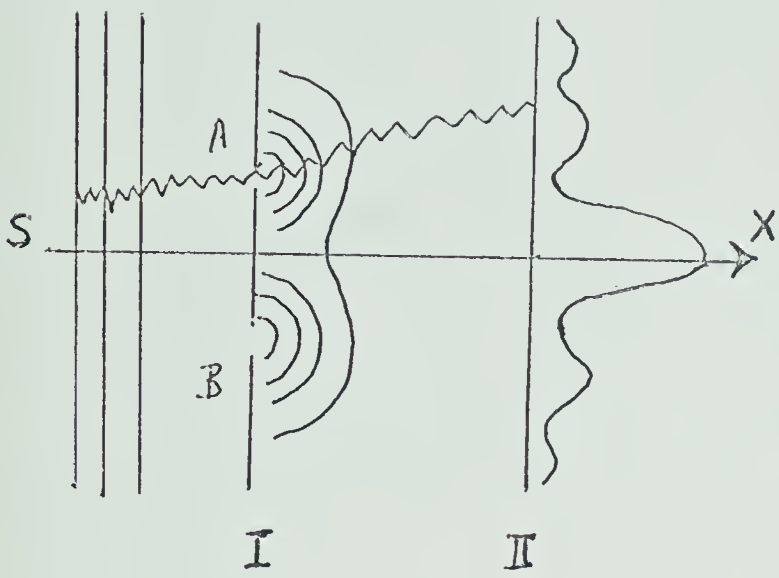

Fig. 2

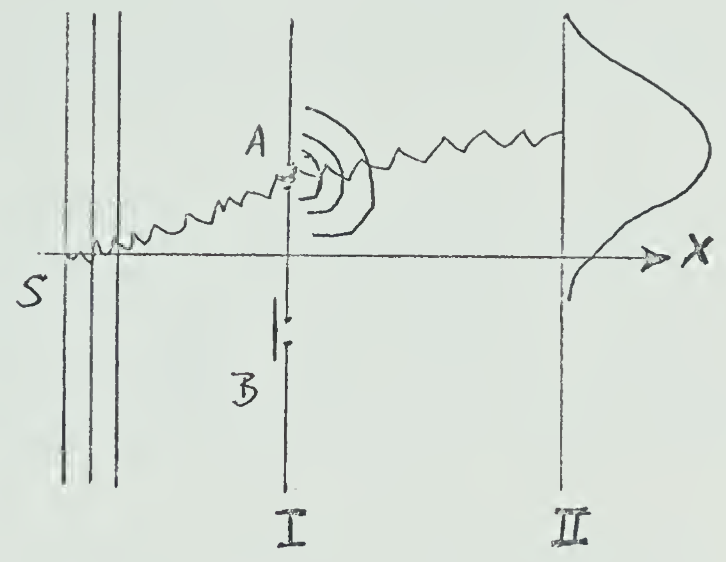

Fig. 3

The associated wave with the body will as before be diffracted through the two slits A and B of screen I. That is, the wave-aspect of the electron passes through both slits and a distribution pattern of high and low intensity will be obtained on screen II. But the particle-aspect of the electron--the body associated with the wave--undergoes a random motion following an irregular pattern and eventually passes through one slit, either A or B. To the right of screen I the body is influenced by the diffracted wave and ends up at a certain point on screen II. After a large ensemble of electrons have passed through screen I, a statistical pattern of the impact-density is obtained which is proportional to the field intensity $|\psi|^2$. " The statistical tendency to appear where $|\psi|^2$ is greatest is due to the effects of the 'quantum force' while the random motions explain why the precise points at which the various particles appear fluctuate in an irregular way." (57) (c.f.Fig. 2)

If we close slit B for example, then the "quantum mechanical" forces will be affected since the wave ceases to have strong and weak fringes. Therefore the closing of slit B influences the particle even though it passes through slit A. (c.f. Fig. 3)

This particular interpretation of quantum theory provides us therefore with a rational explanation of the wave-particle duality phenomenon since it assumes (i) that the <u>same</u> micro-object "X" possesses <u>simultaneously</u> and <u>objectively</u> <u>both</u> dispositional properties to be <u>wave-like</u> and <u>particle-like</u> and (ii) that there is a categorical basis (e.g. sub-quantum mechanical level; hidden variables) that makes "X" (e.g. X = electron) exhibit its <u>wave-like</u> or <u>particle-like</u> property, depending on the mutually exclusive experimental arrangements.

The usual interpretation of quantum theory does not offer us any understanding of the wave-particle duality phenomenon. As Bohm remarks:

> All that we can do is to accept without further discussion the fact that electrons enter the slit system, and appear at the screen with an interference pattern. As to how this came about, such a question cannot even be raised within the framework of the usual interpretation. (58)

CONCLUDING REMARKS

We have demonstrated that a Realist's account of dispositional properties represents a good working hypothesis for scientific research. It supports the methodology behind the line of research undertaken by Bohm and others who find the orthodox interpretation of quantum theory inadequate for physical, methodological as well as epistemological reasons.

Although the new interpretations do suffer from a number of inadequacies which we did not discuss in detail, they provide a new and broader conceptual structure which enables us to raise questions that cannot even be formulated within the conceptual framework of the orthodox interpretation of quantum theory.

Thus the completeness of quantum theory cannot really be questioned, since as we saw, the basic assumptions on which the usual interpretation of the theory rests imply that quantum theory must be assumed to be complete, or as complete as it can ever be.

Therefore, the E.P.R. paradox, e.g., does not arise within the conceptual framework of the orthodox interpretation of quantum theory, because it is assumed that the composite system of two particles that once interacted but are now separated, must be regarded as an indivisible whole. Assumption A is therefore untenable since we cannot treat the once interacting but now separated systems as two independent systems.

But what really *does* happen *physically*, when in such a situation we make a measurement--upon which the state-vector of the composite system collapses into an eigenvector of the composite system ?

Is it advantageous for the theory that according to it, it is impossible to carry out a measurement on only one particle of a system of two particles regardless of how far they are apart, because the composite system must always be regarded as an indivisible whole?

The point is that quantum theory, by the very nature of its assumptions, is constructed in such a way that it excludes the construction of the E.P.R. paradox as a "paradox" from the very outset. But this, we claim, does not constitute a strength of quantum theory but a weakness since it is unable to resolve the paradox in the terms in which the paradox is presented. It is unsatisfactory to simply assert, as does Bohr, that the two separated systems and the measuring apparatus constitute a complete inseparable unanalysable totality, and that accordingly the question as to the origin of the correlations, in the spin problem for example, has no meaning and that such a question should not even be raised.

It is true that at the quantum mechanical level the interaction between measurement apparatus and the system observed is such that the two systems become entangled in an intricate and complex fashion and constitute a combined system. But we disagree with Bohr in that we think it possible, as Bohm points out, "that this combined system is at least conceptually analyzable into components which satisfy appropriate laws" (59), that is, laws of a deeper sub-quantum-mechanical level which approach quantum mechanical laws as an approximation.

We agree with Bohm that the E.P.R. paradox can be understood with the aid of a hidden variable theory " in a perfectly rational way, in terms of a new notion of coordinated fluctuations arising in the sub-quantum-mechanical level." (i.e. categorical basis) (60)

Moreover, we support Bohm and others who propose to go beyond the present limitation of knowledge implied by the orthodox interpretation of quantum theory for we believe, as Karl Jaspers says, that "it is incumbent upon us really to know how much we can know, if we are to attain to true nonknowledge." (61)

At the same time we must guard against what Jaspers calls "scientific superstition" (Wissenschaftsaberglaube) (62) and realize that there may be limitations to scientific methods and their applications. With Bohm, we do not think therefore of hidden variable theories as a return to a Laplacian determinism, recognizing the infinitely complex structure of matter which necessitates, that in all domains of Nature we have a mixture of causal and statistical laws. But we also do not think that the present laws of quantum theory are final and represent an ultimate limit to our knowledge. In fact, to make such a claim seems to us to be a kind of Wissenschaftsaberglaube itself.

REFERENCES AND FOOTNOTES

Chapter I

(1) A. Einstein, B. Podolsky and N. Rosen: Phys. Rev. 47, 777 (1935)

(2) Ibid., 777-778

(3) Ibid., 778

(4) c.f.discussion of Schrödinger representation versus Heisenberg representation, p. 61

(5) A. Einstein, B. Podolsky and N. Rosen: Phys. Rev. 47, 780, (1935)

Chapter II

(1) W. H. Furry: Phys. Rev. 49, 393 (1936)

(2) "We accordingly label a measurement as retrospecive or predictive according as it yields information about the immediately preceeding or immediately suceeding state of the measured system. Of course it is possible for a measurement to be both retrospective and predictive." E.C. Kemble: The Fundamental Principles of Quantum Mechanics, 323

(3) A. Messiah: Quantum Mechanics, 143

(4) Ibid. 197

(5) W. H. Furry: Phys Rev. 49, 394 (1936)

(6) Ibid, 395

(7) Ibid., 395

(8) Ibid., 395

(9) Ibid., 395-6

(10) Ibid., 396
(11) Ibid., 396
(12) Ibid., 396
(13) Ibid., 396
(14) Ibid., 396
(15) Ibid., 396 - 7

Chapter III

(1) E. Schrödinger: Naturwissenschaften, 23, 807-812 (Part I), 823-828 (Part II), 844-849 (Part III) and Proc. Camb. Phil. Soc., 31, 555 (1935)
(2) W. H. Furry: Phys. Rev. , 496 (1936)
(3) D. Bohm, Y. Aharanov: Phys. Rev. 108, 1971 (1957)
(4) A. Einstein: Albert Einstein: Philosopher-Scientist, 85
(5) C. A. Hooker: The Nature of Quantum Mechanical Reality: Einstein versus Bohr, unpublished manuscript, 74
(6) D. Bohm, Y. Aharanov: Phys. Rev., 108, 1070-1
(7) C. S. Wu: Phys. Rev., 77, 136 (1950)
(8) D. Bohm: Quantum Theory, 616
(9) D. Bohm, Y. Aharanov: Phys. Rev., 108, 1070
(10) Ibid., 1071
(11) Ibid., 1071
(12) Ibid., 1071
(13) W.V.O. Quine: The Ways of Paradox, 3
(14) C. A. Hooker: The Nature of Quantum Mechanical Reality: Einstein versus Bohr, unpublished manuscript, 57

(15) Ibid., 57
(16) Ibid., 263
(17) Ibid., 58
(18) D. Bohm, Y. Aharanov: Phys. Rev., 108, 1071 (1957)
(19) Ibid., 1071
(20) W. H. Furry: Phys. Rev., 49, 397 (1935)
(21) C. A. Hooker: The Nature of Quantum Mechanical Reality: Einstein versus Bohr, unpublished manuscript, p. 71
(22) D. Bohm, Y. Aharanov: Nuovo Cimento, 17, 964 (1960)
(23) Ibid., 975
(24) C. A. Hooker: p. 81
(25) Ibid., p. 81
(26) A. Einstein, Podolsky and Rosen: Phys. Rev., 47, (777) (1935)
(27) Ibid., 779
(28) W. H. Furry: Phys. Rev. 49 476 (1936)
(29) Ibid., 476
(30) Ibid., 476
(31) Ibid., 476
(32) C. A. Hooker: p. 76
(33) A. Einstein, Podolsky and Rosen: Phys. Rev., 47, 777 1935
(34) Ibid., 780
(35) E. Schrödinger: Naturwissenshaften, 23, 807 (1935)
E. Schrödinger, Proc. Camb. Phil. Soc., 31, 555 (1935)

Chapter IV

(1) W. H. Furry: Phys. Rev. 49, 496, (1936)
(2) A. Einstein: Journ. of the Franklin Institute, 221, 315-318 (1936)
(3) Ibid., 315

(4) E. Schrödinger: Narurwissenschaften, 48, 810 (1935)
(5) Ibid., 810
(6) K. Popper: "Quantum Mechanics without 'The observer'" in Quantum Theory and Reality, 7
(7) Ibid., 7
(8) Ibid., 15
(9) Ibid., 17
(10) L. E. Ballentine: Rev. Mod. Phys., 42, 358 (1970)
(11) Ibid., 360
(12) Ibid., 360
(13) D. Bohm: "A proposed explanation of quantum theory in terms of hidden variables at a sub-quantum-level." in Observation and Interpretation, 33
(14) E. Schrödinger: Naturwissenschaften, 49, 813 (1935)
(15) Ibid., 832
(16) Ibid., 823
(17) Ibid., 823
(18) Ibid., 823
(19) Ibid., 823
(20) Ibid., 824
(21) Ibid., 824
(22) Ibid., 824
(23) Ibid., 824
(24) Ibid., 824
(25) Ibid., 824
(26) Ibid., 825
(27) Ibid., 825
(28) Ibid., 826
(29) Ibid., 826
(30) Ibid., 827
(31) Ibid., 827
(32) Ibid., 827
(33) E. Schrödinger: Proc. Camb. Phil. Soc., 31, 555, (1935)

(35) The formalism presented below is essentially based on Mittelstadt's representation in Philosophische Probleme der Modernen Physik

Chapter V

(1) D. Bohm: Phys. Rev., 85, 166 (1952)
(2) Ibid., 166
(3) D. Bohm: Causality and Chance in Modern Physics, 85
(4) D. Bohm: Phys. Rev., 85, 167, (1952)
(5) Ibid., 167
(6) Ibid., 167
(7) Ibid., 167
(8) Ibid., 167
(9) Ibid., 168
(10) D. Bohm: Casuality and Chance in Modern Physics, 94
(11) D. Bohm: Phys. Rev., 85, 168 (1952)
(12) Ibid., 168
(13) Ibid., 168
(14) D. Bohm: Causality and Chance in Modern Physics, 86
(15) D. Bohm: Phys. Rev., 85, 187 (1952)
(16) Ibid., 187
(17) Ibid., 187-8
(18) V. Capasso, D. Fortunato and F. Selleri: Revista del Nuovo Cimento, 2, 153 (1970)
(19) J. M. Jauch and C. Piron,: Helv. Phys. Acta, 36, 827 (1965)
(20) J. S. Bell: Rev. Mod. Phys. 38, 447 (1966)
(21) D. Bohm, J. Bub: Rev. Mod. Phys., 38, 453, (1966)
(22) D. Bohm: Causality and Chance in Modern Physics, 95
(23) Ibid., 80
(24) Bohm remarks: ". . . that fields have usually manifested themselves as waves in physics." Ibid., 96

(25) Ibid., 96

(26) Ibid., 96

(27) Ibid., 96

(28) W. Heisenberg: The physical Principles of the Quantum Theory

(29) C. F. von Weizsäcker: Zum Weltbild der Physik, 105

(30) D. Bohm: Causality and Chance in Modern Physics, 97

(31) Immanuel Kant: Critique of Pure Reason, p. 41-3

(32) A. J. Ayer: Language, Truth and Logic, 5-16

(33) K. Popper: The Logic of Scientific Discovery, expecially p. 78-82

(34) D. Bohm: Phys. Rev. 85, 169, (1952)

(35) D. Bohm: Causality and Chance in Modern Physics, 103

(36) Ibid., 104

(37) D. Bohm: Phys. Rev., 85, 166 (1952)

(38) D. Bohm: Causality and Chance in Modern Physics, 106

(39) D. Bohm: Phys. Rev., 85, 188, (1952)

(40) D. Bohm: Causality and Chance in Modern Physics, 92

(41) D. M. Armstrong in A Materialist Theory of Mind, 85 makes that distinction and uses the term "categorical basis". However, our treatment of dispositional properties is more extensive and differs in several points from the interpretation given by Armstrong.

(42) Scientific American, Sept. 1967, p. 222

(43) Quine: Word and Object, 223

(44) N. Goodmann: Fact Fiction and Forecast, p 40

(45) K. Popper: The Logic of Scientific Discovery, 424

(46) Quine: Word and Object, 223

(47) K. Popper: The Logic of Scientific Discovery, 425

(48) R. Carnap: "Testability and Meaning," Minnesota Studies in Philosophy of Science, p. 53-54

(49) Bohm remarks: "Actually of course all waves must consist of wave packets, but in this case, the packet is so much bigger than the slits that we can approximate it as in infinite plane wave." Causality and Chance in Modern Physics, 115

(50) D. Bohm, J. Bub: Rev. Mod. Phys., 38, 456 (1966)

(51) D. Bohm: Causality and Chance in Modern Physics, 111-116

(52) V. Capasso, D. Fortunato and F. Selleri: Revista del Nuovo Cimento, 2, 175-6 (1970)

(53) D. Bohm: Causality and Chance in Modern Physics, p. 113

(54) Ibid., 113

(55) Ibid., 113

(56) Ibid., 114

(57) Ibid., 115

(58) Ibid., 116

(59) D. Bohm, Y. Aharanov: Phys. Rev., 108, 1072 (1957)

(60) Ibid., 1072

(61) K. Jaspers: Philosophie, Vol. II, 262

(62) Ibid., 262

BIBLIOGRAPHY

(All translations from German texts cited in this thesis are my own.)

Armstong, D. M., A Materialist Theory of the Mind. Routledge and and Kegan Paul Ltd., 1968.

Ayer, A. J., Language, Truth and Logic. Dover Publications, Inc.

Ballentine, L. E., "The Statistical Interpretation of Quantum Mechanics", Reviews of Modern Physics, 42 (1970) 358-381.

Bell, J. S., "On the Einstein Podolsky Rosen Paradox", Physics, 1 (1964), 195-200.

__________. "On the Problem of Hidden Variables in Quantum Mechanics", Reviews of Modern Physics, 38 (1966), 447-475.

Bohm, D., Quantum Theory. Prentice-Hall, N.J.

__________. "A Suggested Interpretation of the Quantum Theory in Terms of 'Hidden Variables I'", Physical Review, 85 (1952), 166-179.

__________. "A Suggested Interpretation of the Quantum Theory in Terms of 'Hidden Variables II'", Physical Review, 85 (1952), 180-193.

__________. "Proof that the Probability Density Approaches in Causal Interpretation of the Quantum Theory", Physical Review, 89 (1953), 459-466.

__________. Causality and Chance in Modern Physics. Harper and Row, N. Y., 1961.

Bohm, D., Aharanov, Y., "Discussion of Experimental Proof for the Paradox of Einstein, Rosen and Podolsky", Physical Review, 108 (1957), 1070-1076.

Bohm, D., Bub, J., "A Proposed Solution of the Measurement Problem in Quantum Mechanics by a Hidden Variable Theory", Review of Modern Physics, 38 (1966), 453-469.

Bohm, D., Bub, J. "A Refutation of the Proof by Jauch and Piron that Hidden Variables can be Excluded in Quantum Theory", Review of Modern Physics, 38 (1966), 470-475.

Bohr, N. "Can Quantum-Mechanical Description of Physical Reality be Considered Complete?", Physical Review, 48 (1935) 696-702.

__________. "Causality and Complementarity", Philosophy of Science, 4 (1937), 289-298

__________. Atomic Physics and Human Knowledge. Wiley, N.Y. (1958)

__________. Atomic Theory and the Description of Nature. Cambridge U. P., Cambridge, (1961).

__________. Essays 1958/62 on Atomic Physics and Human Knowledge. Interscience, N. Y. (1963).

__________. "Discussion with Einstein on Epistemological Problem in Atomic Physics", in Albert Einstein: Philosopher-Scientist. The Library of Living Philosophers Inc., Volume VII, First Edition, Cambridge University Press, London, (1949).

Bopp, F. (Ed.) Werner Heisenberg und die Physik unserer Zeit. Friederich Vieweg, Braunschweig, (1961).

Breitenberger, E. "On the So-Called Paradox of Einstein, Podolsky and and Rosen", Il Nuovo Cimento, XXXCIII (1965), 356-360.

Broglie, L. de "The Reinterpretation of Wave Mechanics", Foundation of Physics, 1 (1970), 5-15.

Bub, J., "What is a Hidden Variable Theory of Quantum Phenomena?", International Journal of Theoretical Physics, 2 (1969), 101-124.

Buchdahl, G., Metaphysics and the Philosophy of Science. Basil Blackwell, Oxford, (1969).

Bunge, M., Causality. Meridian Books, N. Y., (1959).

__________. The Critical Approach : Essays in Honour of Karl Popper. The Free Press of Glencoe, N. Y. (1964).

__________. "Physics and Reality", Dialectica, 20 (1966), 144-195.

__________. Foundations of Physics. Springer-Verlag, N.Y., (1967).

__________. *Scientific Research.* Volumes I and II. Springer-Verlag, N.Y., 1967.

__________. *Quantum Theory and Reality.* Springer-Verlag, N.Y., 1967.

__________. *The Delaware Seminar in the Foundations of Physics.* Vol. I, Springer-Verlag, N.Y., 1967.

Burger, J. M., "The Measuring Process in Quantum Theory", *Reviews of Modern Physics, 35* (1963), 145-50.

Capasso, V., Fortuanots, D. and Selleri, F., "Von Neumann's Theorem and Hidden-Variable Models", *Revista del Nuovo Cimento, 2* (1970) 149-98.

Carnap, R., "Testability and Meaning", *Readings in the Philosophy of Science.* eds. H. Feigl and M. Brodbeck, New York: Appleton, Century, Crofts, 1953.

Cassirer, E., *Determinism and Indeterminism in Modern Physics.* Yale U. P., New Haven, Conn., 1956.

Cohen, R. S., Wartofsky, M. W., *Proceedings for the Boston Colloquium in the Philosophy of Science 1962-1964.* Humanities Press, N.Y., 1965.

Cohen, R. S., *Proceedings for the Boston Colloquium in the Philosophy of Science 1966-68.* Humanities Press, N.Y., 1969.

Colodny, R. G., *Frontiers of Science and Philosophy.* Vol. I of the University of Pittsburgh Series in the Philosophy of Science, University of Pittsbirgh Press, Pittsburgh, 1962.

Colodny, R. G., *Beyond the Edge of Certainty.* Vol. II of the University of Pittsbirgh Series in the Philosophy of Science. University of Pittsburgh Press, Pittsburgh, 1965.

Dirac, P. A. M., *Principles of Quantum Mechanics.* Clarendon Press, Oxford, 1967.

Einstein, A., Podolsky, B., Rosen, N., "Can Quantum Mechanical Description of Physical Reality be Considered Complete?" *Physical Review, 47* (1935), 777-780.

Einstein, A., "Remark fo the Essays Appearing in this Collective Volume", in *Albert Einstein: Philosopher-Scientist.* The Library of Living Philosophers Inc., First Edition, Cambridge University Press, London, 1949.

________. "Physik Und Realität", <u>Journal of the Franklin Institute.</u> <u>221</u>, (1936), 313-347.

Feigl, H., Maxwell, A., (Eds.) <u>Current Issues in the Philosophy of Science</u>. Holt, Rinehart and Winston, N.Y., 1961.

Feyerabend, P. K., "Realism and Instrumentalism: Comments on the Logic of factual Support" in <u>The Critical Approach: Essays in Honour of Karl Popper</u>. The Free Press of Glencoe, N.Y., 1964.

________. "On a Recent Critique of Complementarity", <u>Philosophy of Science</u>: Part I, <u>35</u> (1968) 309-331. Part II, <u>36</u> (1969) 82-105.

Furry, W. H., "Note on the Quantum-Mechanical Theory of Measurement", <u>Physical Review</u>. <u>49</u> (1936), 393-399.

________. "Remark on Measurements in Quantum Theory", <u>Physical Review</u>, <u>49</u> (1936), 476.

Goodman, N., <u>Fact, Fiction and Forecast</u>, Athlone Press, London, 1954.

Heelan, P., <u>Quantum Mechanics and Objectivity</u>. Martinus Nijhoff, The Hague, 1965.

Heisenberg, W., <u>The Physical Principles of the Quantum Theory</u>. Dover, N. Y., 1930.

________. <u>Philosophical Problem of Nuclear Science</u>. Faber and Faber, London, 1952.

________. <u>Physics and Philosophy</u>. Harper Torchbooks, N.Y., 1966

Hooker, C. A., "Concerning Einstein's, Podolsky's and Rosen's Objection to Quantum Theory", <u>Americal Journal of Physics</u>. <u>38</u> (1970), 851-857.

________. "The Nature of Quantum Mechanical Reality: Einstein versus Bohr", to be published in: Colodny, R. G. (Ed.) <u>Paradigm and Paradoxes: The Philosophical Challenge of the Quantum Domain</u>. Vol. V of the University of Pittsburgh Series in the Philosophy of Science, University of Pittsburgh Press, Pittsburgh, 1971.

Inglis, D.R., "Completeness of Quantum Mechanics and Charge-Conjugation Correlations of Theta Particles", <u>Reviews of Modern Physics</u>, <u>33</u> (1961), 1-7.

Jammer, M., *The Conceptual Development of Quantum Mechanics.* McGraw-Hill N.Y., 1966.

Jasper, K., *Philosophie.* Vol. II, 3rd. Ed., Berlin: J. Springer, 1956.

Jauch, J., Piron, C., "Can Hidden Variables be Excluded in Quantum Mechanics?", *Helvetia Physica Acta.* 36 (1967), 827-837.

Jauch, J., *Foundations of Quantum Theory.* Addison-Welsey, N.Y., 1968.

Kant, I., *Critique of Pure Reason.* Tranl. by N. Kemp Smith, Macmillan & Co. Ltd., 1964.

Kemble, E., *Fundamental Principles of Quantum Mechanics.* Dover edition, 1958.

Körner, S. (Ed.) *Observation and Interpretation of Physics.* Dover, N. Y., 1962.

Krips, H.P., "Two Paradoxes in Quantum Theory", *Philosophy of Science,* *36* (1969), 145-152.

Kuhn, T., *The Structure of Scientific Revolutions.* University of Chicago Press, Chicago, 1962.

Lange, H., *Geschichte der Grundlagen der Physik.* Karl Alber Freiburg, München, 1954.

__________. *Geschichte Der Grundlagen der Physik.* Karl Alber Freiburg, München, 1961.

Margenau, H., "Quantum-Mechanical Description", *Physical Review.* *49* (1936), 240-242.

Mittelstadt, P., *Untersuchungen zur Quantenlogik.* Verlag der Bayerischen Akademie der Wissenschaften, München, 1959.

Nagel, E., *The Structure of Science.* Harcourt, Brace & World, Inc. 1961.

__________. *Philosophische Probleme der Modernen Physik.* Bibliographisches Institut, Mannheim, 1968.

Paulie, W., *Aufsätze und Vorträge über Physik und Erkenntnistheorie.* Friedr. Vieweg & Sohn, Braunschweig, 1961.

Peares, A., Singer, P., "On Possible Experimental Tests for the Paradox of Einstein, Podolsky and Rosen", Il Nuovo Cimento. XV (1960), 907-915.

Popper, K., The Logic of Scientific Discovery. Harper & Row, New York 1965.

__________. Conjectures and Refutations. Harper & Row, New York, 1968.

__________. "Quantum Mechanics without 'The Observer'", in Bunge, M., Quantum Theory and Reality, Springer-Verlag, N.Y., 1967.

Putnam, H., "Comments on the Paper of David Sharp", Philosophy of Science. 28, (1961), 234-237.

__________. "A Philosopher Looks at Quantum Mechanics" in Beyond the Edge of Certainty. Vol. II of the University of Pittsburgh Series in the Philosophy of Science, University of Pittsburgh Press, Pittsburgh, 1965.

Quine, W. V. O., From a logical Point of View. Harper and Row, N.Y., 1963.

__________. Word and Object. M. I. T. Press, Massachusetts, (1964).

__________. Ways of Paradox. Random House, New York, (1966).

Reichenbach, R., Philosophical Foundations of Quantum Theory. University of California Press, Los Angeles, (1944).

Sachs, M., "On Pair Annihilation and the Einstein-Podolsky-Rosen Paradox", International Journal of Theoretical Physics. 1 (1968). 387-407.

Scheibe, E., Die Kontingenten Aussagen in der Physik. Athenäum Verlag, Frankfurt am Main, (1964).

Schiff, H. I., Quantum Mechanics. McGraw-Hill, (1968).

Schlegel, R. "Statistical Explanation in Physics: The Copenhagen Interpretation", Synthese. 21 (1979), 65-82.

Schrödinger, E., "Die Gegen wärtige Situation in der Quantum Mechanik", Natur wissen schaften. 23 (1935), 807-812, (Part I): 823-828 (Part II); 844-849 (Part III).

________. "Discussion of Probability Relations Between Separated Systems", Proceedings of the Cambridge Philosophical Society. 31 (1935), 555-563.

________. "Probability Relations Between Separated Systems, Proceedings of the Cambridge Philosophical Society. 32 (1936), 446-452.

________, Was ist ein Naturgesetz? R. Oldenbourg München, Wein, (1967).

________. Geist und Materie. Friedr. Vieweg & Sohn. Braunschweig, (1965).

________. "Are there Quantum Jumps?" British Journal for the Philosophy of Science. 3 233-242 (Part II).

Sharp, D. "The Einstein-Podolsky-Rosen Paradox Re-Examined", Philosophy of Science. 28 (1961), 225-233.

Titze, H. Der Kausalbegriff in Philosophie und Physik. Verlag Anton Hain, Meisenheim am Glan, (1964).

Vigier, J. P. "The Concept of Probability in the Frame of the Probabilistic and the Causal Interpretation of Quantum Mechanics", In Körner, S. (Ed.) Observation and Interpretation in the Philosophy of Physics. Dover, N.Y. (1962).

________. "Hidden Parameters Associated with Possible Internal Motions of Elementary Particles". In Bunge, M. Quantum Theory and Reality. Springer-Verlag, N.Y., (1967).

________. "Possible Internal Subquantum Motions of Elementary Particles", in Yourgrau, W., Breck, A.D. (Eds.) Physics, Logic and History. Plenum Press, N. Y. (1970).

Weizsäcker, C. F. von, Zum Weltbild Der Physik, S. Hirzel Verlag Stuttgart, (1970).

Weyl, H. Philosophie der Mathematik und Natur wisseuschaft. R. Oldenbourg München. Wien, (1966).

________. Gruppen theorie und Quanten Mechanik. Hirzel, Stuttgart, (1967).

Wigner, E. P. "Physics and the Explanation of Life". Foundation of Physics, 1 (1970), 35-45.

Wigner, E. P. "The Problem of Measurement", American Journal of Physics. 31 (1963).

________. "On Hidden Variables and Quantum Mechanical Probabilities", American Journal of Physics. 38 (1970), 1005-1009.

Wu, C. S., Shaknov, I., "The Angular Correlation of Scattered Annihilation Radiation". Physical Review. 77 (1950), 136.